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Assessing the structure dependence between the Spanish stock market and some international financial markets. A time-varying copula analysis.

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Abstract: In this study we use time-varying copula analysis to investigate the dependence between the Spanish stock market, represented by the IBEX35 index, and some international stocks and commodities markets. The results indicate that: first, the European stock markets offer limited diversification possibilities. Second, American markets offer higher diversification possibilities than the European markets but the diversification may not work in an extreme market condition; here we find strong evidence of contagion effect. Third, the Asian markets outperform to the American markets offering higher diversifications possibilities even in extreme market conditions. Fourth, the assets negotiated in the Shanghai market may be considered hedge assets instead of diversifier assets; this feature is shared by the Bitcoin and Gold although the role of this last asset is highly volatile. These results provide useful information for those who seek to actively diversify their international portfolios and to manage their worldwide assets. Finally, we observe that degree of dependence derived from the correlation analysis is notably higher than the suggested by the copula analysis; this may be due the fact that correlation coefficient does not consider conditional heteroscedasticity so that correlations will be biased upwards.

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PALABRAS CLAVE:
Dependencia, Efecto contagio, Copula, Diversificación, Cobertura

Resumen: En este estudio utilizamos cópulas dinámicas para investigar la dependencia entre el mercado de valores español, representado por el índice IBEX35, y algunos mercados internacionales de acciones y materias primas. Los resultados indican que: en primer lugar, las bolsas europeas ofrecen posibilidades de diversificación limitadas. En segundo lugar, los mercados estadounidenses ofrecen mayores posibilidades de diversificación que los mercados europeos, pero es posible que la diversificación no funcione en condiciones de extremas de mercado; en esos casos encontramos una fuerte evidencia del efecto de contagio. En tercer lugar, los mercados asiáticos superan a los mercados estadounidenses y ofrecen mayores posibilidades de diversificación incluso en condiciones de mercado extremas. En cuarto lugar, los activos negociados en el mercado de Shanghái pueden considerarse activos de cobertura en lugar de activos diversificadores. Esta característica es compartida por el Bitcoin y el oro, aunque el papel de este último activo es altamente volátil. Estos resultados brindan información útil para aquellos que buscan diversificar activamente sus carteras internacionalmente y administrar sus activos en todo el mundo. Finalmente, observamos que el grado de dependencia derivado del análisis de correlaciones es notablemente superior al sugerido por el análisis de copula; esto puede deberse a que el coeficiente de correlación no considera la heterocedasticidad condicional, por lo que las correlaciones estarán sesgadas al alza.

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1. Introduction

The literature on dependence between financial markets and the existence of *contagion effect*[1] is quite extends due to its implications on asset allocation and portfolio diversification (He et al. 2015). To assess this subject, different methodologies have been used. The first's studies applied correlation analysis; by using this measure, the *contagion effect* is detected when a statistically significant increase in correlation is observed during the crash. For example, King and Wadhwani (1990) find evidence of an increase in stock returns correlation in 1987 crash. Calvo and Reinhart (1996) report correlation shifts during the Mexican Crisis, while Baig and Goldfajn (1999) support the contagion phenomenon during the East Asian Crisis. Hon et al. (2007) find that technology bubble collapse in the US resulted in an increase in correlation between the US and other foreign stock markets. Soon afterwards, some authors argued that tests for contagion based on the correlation coefficient were inadequate. The correlation coefficient does not consider conditional heteroscedasticity so that during a crisis, cross-market correlations are biased upwards[2].

To overcome the limitations of correlation methods, researchers have used Dynamic Conditional Correlation (DCC) (see Case et al. 2012, Yang et al. 2012, and Rong and Trück, 2014 among others). This methodology has the advantage of taking conditional heteroscedasticity into account and of allowing also to calculate the dynamic conditional correlation between the series. The main drawback of this method is that it requires the normal assumption for the multivariate distribution; namely, this model might not provide an appropriate measure of the dependence among financial markets when the multivariate normality assumption on the joint distribution of the data set does not hold.

Both methods allow us to identify *dependence* and *contagion* effect when there exists linear relationship between the marginals or series under the study. However, when the relationship between marginals is non-linear these models will not be able to reap correct results. To overcome the limitations of these methodologies a copula analysis has been proposed. Unlike the aforementioned methods, the copula analysis can explain non-linearity relationship between marginals without the constraint of normality. Furthermore, the copula analysis provides information on both the degree of correlation and dependence structure (Reboredo, 2011). Many papers study dependence between stocks markets and exchange markets based on copula analysis, see for instance Meucci (2010); Samitas and Tsakalos (2013) and Syriopoulos and Roumpis (2009).

Applications of copula analysis on dependence and *contagion* effects between international stocks markets can be found in Rajwani et al. (2019), Wang et al. (2011), Horta et al. (2010), Hussain et al (2018), Wen et al. (2012), Nguyen et al. (2017), Das (2016), Changquing et al. (2015), Kenourgios et al. (2010) and Jondeau and Rockinger (2006) among others. Most of these papers analyze dependence between the US stock market and other international stocks markets; the dependence between Asian stock markets has been extensively studied as well.

Rajwani et al. (2019) use copula analysis for studying the dependence between the US stock market and Asian stock markets, namely China, Hong Kong, Indonesia, South Korea, Malaysia, Japan, India and Taiwan. They conclude that the dependence with Asian markets is not high although it increases in crisis periods, indicating a *contagion effect* due to the sub-prime crisis with epicenter in USA; the lowest dependence is found for China market. Horta et al. (2010) adopt the copula methodology to assess financial contagion from the US subprime crisis to four European stock markets: Portugal, France, Belgium and The Netherlands. For all markets considered, they find that the dependence structure increases in crisis periods, although the country less affected was Portugal. They also test whether the increase of dependence in crisis period was statistically significant, finding evidence of that. Nguyen et al. (2017) make use of copula functions to empirically examine the left tail dependence between the US stock market and stock markets in Vietnam and China, this last country represented by both Shanghai index and Shenzhen index. The highest left tail dependence is found between US market and Vietnan market. The US and Shanghai stock markets exhibit left tail dependence before the crisis, but no evidence of post-crisis tail dependency. On the contrary, the Shenzhen stock market is independent of the US market before and after the crisis which implies that an extreme event in the US market is less likely to influence the Shenzhen stock market. Hence, for this combination, there is a significant potential risk of diversification if US investors invest in the Shenzhen market. Wang et al. (2011) study the dependence structure between China's stock market and other world markets as US market, Europe markets, Japan market and Pacific Asian markets. The highest dependence is found between China and Asian Pacific markets followed by China with Japan market. The lowest dependence is found between China and USA followed by China and Europe markets. Hussain et al (2018) use EVT (Extreme Value Theory) and copula analysis to investigate the dependence between pairs of China economic area stock markets. In particular, four markets are considered: Shanghai, Shenzhen, Hong Kong and Taiwan. The dependence between the Shanghai market and Shenzhen market is strongest followed by the Hong Kong and Taiwan markets. The weakest dependence is found between Taiwan and Shanghai markets and between Taiwan and Shenzhen markets. This means that the diversification may be more effective for these two last markets. Wen et al. (2012) apply copula analysis to investigate whether a *contagion effect* exists between energy and stock markets during the recent financial crisis. Using the WTI oil spot price, and three stock indexes: S&P500(USA), SHCI (Shanghai) and SZHI (Shenzhen), evidence was found for a significantly increasing dependence between crude oil and stock markets after the failure of Lehman Brothers, thus supporting the existence of contagion in the sense of Forbes and

[1] By contagion effect we understand a significant increase in cross-market correlations between any two markets from pre-crisis period to crisis period (Forbes and Rigobon, 2002).

[2] Forbes and Rigobon (2002) used a numerical example to show that linear correlation coefficients are conditional on volatility and are biased upwards in periods of crisis. As a consequence, assessments that do not take such bias into account may mistakenly report evidence of contagion in cases where correlation coefficients simply pick up the high levels of co-movement, or interdependence, existing between the analysed countries in moments of financial turmoil, but also in calm periods.

Rigobon's (2002) definition. Das (2016) examines the dependence of the India stock market and other major Asian markets of China, Hong Kong, Japan and Taiwan. The study reveals that the Indian market keeps a high dependence with Hong Kong and Japan market, followed by Taiwan. The lowest dependence is found with China market. Besides, it presents that the low tail dependence was higher than the upper tail dependence and both of them were higher than the linear correlation.

Our study is framed in this area as we investigate the dependence structure between the Spanish stock market, represented by the IBEX35 Index, and some international financial markets including stock markets and commodities markets. As the dependence structure may change along the time we conduct this study in three sub samples: (i) Before the Global Financial Crisis (GFC) that cover the period from 2000 to 2007; (ii) During the GFC, from 2008 to 2010 and (iii) After the GFC, from 2011 to 2019. By studying the dependence in different periods we intend to investigate whether the dependence structure increases significantly in a crisis period, when the diversification is more needed. Besides, this study will give information about the existence of *contagion effect* between financial markets. We use copula analysis which appropriately describes the dependence structure between financial assets (see e.g. Cherubini and Luciano, (2001); Frey and McNeil, (2003); Jondeau and Rockinger, (2006); Junker et al., (2006); and Luciano and Marena, (2002)). Copula approach allows us investigating both the conditional dependence structure and the conditional tail dependence between markets.

The objectives of the study are: *first*, to understand the relationship, if any, of the Spanish stock market with some of the major stock markets around the world and some commodities; *second*, to establish the importance of copula functions with respect to linear correlation coefficient in understanding this relationship; *finally*, to analyse the possibilities of diversification and coverage that these markets offer to investors. This study can provide useful information for those who seek to actively diversify their international portfolios and to manage their worldwide assets.

Our paper contributes to the literature in several dimensions. It is one of the first studies to thoroughly investigate the nonlinear relationship between Spanish stock market and the international financial markets using copulas. Further, the study is very comprehensive since it includes a large number of international stock markets from different geographical areas and some commodities, including the Bitcoin whose characteristics as a *hedge* and *diversifier* asset have been scarcely studied in the relevant literature. To last, the subsample analysis contributes to the literature on *contagion effect* which has not been studied for the Spanish market.

The rest of the paper is structured as follows. First, we describe the methodology used to assess the study. The data set is introduced in Section 3. Section 4 presents the empirical results. Finally, Section 5 concludes this paper.

2. Methodology

There are two main steps in modelling the dependence structure. In the first step we fit an AR (*p*)-APARCH model to the univariate return series and obtain the standardized residuals for each series. In the second step, we use standardized residuals to estimate the different copula functions[3]. In the following lines, we first review the volatility specification APARCH. Then we review the copula models.

2.1. APARCH model

The autoregressive models AR(*p*) can be used for modelling the conditional returns. Having detecting first order correlation in the returns of CAC40, FTSE, S&P500, MERVAL, IBOVESPA, GOLD, SILVER and COPPER we use an AR(1) model for modelling the conditional return of these variables:

$$r_t = \mu + \emptyset r_{t-1} + \varepsilon_t \text{ ; where } \varepsilon_t = \sigma_t z_t \tag{1}$$

In the cases in which first order correlation have not been detected (IBEX35, DAX, IPC, NIKKEI, KOSPI, HSI, SSE, BITCOIN), the model used for modelling conditional return is as follow:

$$r_t = \mu + \varepsilon_t \text{ ; where } \varepsilon_t = \sigma_t z_t \tag{2}$$

For modelling the conditional variance, in this paper we apply the APARCH(1,1) model (Asymmetric Power ARCH model) proposed by Ding, Granger, and Engle (1993). This model can well express volatility clustering, fat tails, excess kurtosis and the leverage effect. The APARCH variance equation is,

$$\sigma_t^\delta = \omega + \alpha_i(|\varepsilon_{t-1}| + \gamma_1\varepsilon_{t-1})^\delta + \beta_1\sigma_{t-1}^\delta \tag{3}$$

Where ω, α_1, γ_1, β_1 and δ are additional parameters to be estimated. The parameter γ_1 reflects the leverage effect ($-1 < \gamma_1 < 1$). A negative (resp. positive) value of γ_1 means that past negative (resp. positive) shocks have a deeper impact on current conditional volatility than past positive (resp. negative) shocks. The parameter δ plays the role of a Box-Cox transformation of σ_t ($\delta > 0$).

The APARCH equation is supposed to satisfy the following conditions, *i)* $\omega > 0$ (since the variance is positive), $\alpha_1 \geq 0$, $\beta_1 \geq 0$. When $\alpha_1 = 0$, $\beta_1 = 0$, then $\sigma^2 = \omega$, *ii)* $0 \leq \alpha_1 + \beta_1 \leq 1$. The APARCH model is a general model because it has great flexibility, having as special cases, among others, GARCH and GJRGARCH models.

[3] Although it is possible to model the dependence structure using original returns some studies point that it is more appropriate to study the dependence structure after filtering out the autoregressive and heteroscedastic behavior of the data (see Gregoire et al. 2008, Jondeau and Rockinger, 2006, and Patton, 2006).

To estimate this model, we have to assume a distribution for the innovations. To assess the distribution that fits best the data we compare several fat tail distribution:

i) Student-t,
ii) Generalized error distribution (GED),
iii) Skew Student-t and skew GED.

Normal distribution is excluded of the comparison because of the strong evidence rejecting the normality distribution in our data set (see Table 2).

Copulas

We derive the dependence structure between two markets via copula examining the dependence between the marginal distribution of the standardized innovations, that we assume ST-AR*(p)*-APARCH(1,1) for S&P500, DAX, KOSPI, SSE, Gold, Silver and GED-AR*(p)*-APARCH for the rest of the assets considered. Bivariate copulas are used in this study.

Let be X_1, X_2 a two random variables with a marginal distribution function given by $F_i(x_i)$ where $F_i(x_i) = \Pr(X_i \leq x_i)$ for $i = 1,2$. A copula is a function that joins, or couples the univariate distribution functions to a multivariate distribution function (*F*), as denoted by *C* in the equation.

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \tag{4}$$

Alternatively, a copula can be defined as the multivariate distribution, *C*, of a vector of random variables with uniformly distributed marginals *U*(0,1)

$$C(u_1, u_2) = F(F_1^{-1}(u_1), F_2^{-1}(u_2)) \tag{5}$$

where the $u_i = F(x_i)$ and F_i^{-1}'s are the quantile functions of the marginals. A copula extracts the dependence structure from the joint distribution, independent of marginal distributions. Deriving the equation (3) the density copula (*c*) is obtained.

$$c(u_1, u_2) = f(f_1^{-1}(u_1), f_2^{-1}(u_2)) \tag{6}$$

In the Table 1, we present the functional forms of the five copulas used in this paper which are the most commonly used in this kind of studies: (i) Gaussian, (ii) Student-t, (iii) Clayton, (iv) Gumbel and (v) Frank. In this table we also include de density copula.

In the case of Gaussian and Student-t copula, ρ is simply the linear correlation coefficient between the two random variables. $\rho = 0$ describes the independence copula, while for $\rho = 1$ describes the comonotonicity copula, and for $\rho = -1$ the countermonotonicity copula. The normal copula is symmetric and does not exhibit tail independence. The Student-t copula has symmetric but nonzero tail dependence and nests the normal copula. The coefficient of tail dependence is given by: $\lambda = 2t_{v+1}\left(\frac{\sqrt{v+1}\sqrt{1-\rho}}{\sqrt{1+\rho}}\right)$.

Table 1: Elliptical and Archimedean copulas

	Functional form of copula	Density copula
Elliptical copulas		
Gaussian copula	$C(u_1, u_2;\ \rho) = \Phi_2(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$	$c(u_1, u_2;\ \rho) = \frac{1}{\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}(x_1^2 - 2\rho x_1 x_2 + x_2^2)\right)$
Student-t copula	$C(u_1, u_2; \rho) = T_{v,}(t_{v_1}^{-1}(u_1), t_{v_2}^{-1}(u_2))$	$c(u_1, u_2;\ \rho) = \frac{K}{\sqrt{1-\rho^2}}\left[1 + \frac{1}{v(1-\rho^2)}(x_1^2 - 2\rho x_1 x_2 + x_2^2)\right]^{\frac{v+2}{2}} [(1 + v^{-1}x_1^2)(1 + v^{-1}x_2^2)]^{\frac{v+2}{2}}$
Archimedean copulas		
Clayton copula	$C(u_1, u_2; \alpha) = (u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-1/\alpha}$	$c(u_1, u_2;\ \alpha) = (1+\alpha)(u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-\frac{1}{\alpha}-2}(u_1 u_2)^{-\alpha-1}$
Gumbel copula	$C(u_1, u_2; \delta) = \exp\{-((-\ln(u_1))^{\delta} + (-\ln(u_2))^{\delta})^{1/\delta}\}$	$c(u_1, u_2; \delta) = (A + \delta - 1)A^{1-\delta} \exp(-A)(u_1 u_2)^{-1}(-\ln u_1)^{\delta-1}(-\ln u_2)^{\delta-1}$
Frank copula	$C(u_1, u_2; \theta) = -\frac{1}{\theta} \ln\left(1 + \frac{(e^{(-\theta u_1)} - 1)(e^{(-\theta u_2)} - 1)}{e^{(-\theta)} - 1}\right)$	$c(u_1, u_{2;\ \theta}) = \frac{\theta[1 - \exp(-\theta)]\exp(-\theta u_1 u_2)}{([1 - \exp(-\theta)] - (1 - \exp(-\theta u_1))(1 - \exp(-\theta u_2)))^2}$

Note: Φ_2 is the standard bivariate normal; ρ is simply the linear correlation coefficientΦ^{-1} is the inverse of the standard univariate Gaussian; T_v is the standard bivariate Student-t with v degrees freedom; $t_{v_i}^{-1}$ is the inverse of the standard univariate Student-t with v_i degree of freedom. In copula Clayton the dependence is measure by α parameter. δ denotes de dependence parameter in copula Gumbel and θ is the dependence parameter in copula Frank. In addition, $K = \Gamma\left(\frac{v}{2}\right)\Gamma\left(\frac{v+1}{2}\right)^{-2}\Gamma\left(\frac{v}{2}+1\right)$; $A = [(-\ln u_1)^{\delta}(-\ln u_2)^{\delta}]^{1/\delta}$

Clayton copula captures independence (α = 0) and perfect positive dependence ($\rightarrow \infty$). This copula has asymmetric tail dependence. The dependence in upper tail is zero ($\lambda_U = 0$) while the dependence in lower tail is given by $\lambda_L = \begin{cases} 2^{-1/\alpha}, & \alpha > 0 \\ 0 & otherwise \end{cases}$.

Gumbel copula captures a large range of dependence from independence (δ = 1) to perfect positive dependence ($\delta \rightarrow \infty$). Gumbel copula has asymmetric tail dependence. The dependence in lower tail is zero ($\lambda_L = 0$) while the dependence in upper tail is given by $\lambda_U = \begin{cases} 2 - 2^{1/\delta}, & \delta > 1 \\ 0 & otherwise \end{cases}$.

Frank copula permits modelling positive as negative dependence in the data $\theta \epsilon(-\infty, +\infty)$. The independence case will be attained when θ approaches zero. However, the Frank copula has neither lower nor upper tail dependence $\lambda_U = \lambda_L = 0$; the Frank copula is thus suitable for modeling data characterized by weak tail dependence.

Table 2 summarizes tail dependence and Kendall's tau for the copulas used in this study.

Table 2: Tail dependence and Kendall's tau for various copulas

	Left tail dependence	**Right tail dependence**	**Kendall's tau**
Normal	0	0	$\tau = \frac{2}{\pi} arc\sin R_{12}$
t-student	$\lambda = 2t_{\upsilon+1}\left(\frac{-\sqrt{\upsilon+1}\sqrt{1-R_{12}}}{\sqrt{1+R_{12}}}\right)$	$\lambda = 2t_{\upsilon+1}\left(\frac{-\sqrt{\upsilon+1}\sqrt{1-R_{12}}}{\sqrt{1+R_{12}}}\right)$	$\tau = \frac{2}{\pi} arc\sin R_{12}$
Gumbel	0	$\lambda_U = 2 - 2^{1/\alpha}$	$\tau = 1 - \frac{1}{\alpha}$
Clayton	$\lambda_L = \begin{cases} 2^{-1/\alpha}, & \alpha > 0 \\ 0 & in\ other\ case \end{cases}$	0	$\tau = \frac{\alpha}{\alpha+2}$
Frank	0	0	$\tau = 1 + \frac{4}{\alpha}\left(\frac{1}{\alpha}\int_0^\alpha \frac{t}{e^t-1}dt\right)$

Theory of (unconditional) copula is extended to the conditional case, thus allowing to use copula theory in the analysis of time-varying conditional dependence. The functional form of the copula remains fixed over the sample whereas the copula parameters vary according to some evolution equation.

To specify the dynamics of the copula dependence parameter, Patton (2006) proposes observation-driven copula models where the time-varying dependence parameter is a parametric function of transformations of the lagged data and an autoregressive term. Using the marginal distributions of the standardized residuals $u_{1,t}$ and $u_{2,t}$ the dynamics of the parameters for the Gaussian, Student-t, Gumbel, Clayton, and Frank copulas can be specified.

For the dynamics of the **Gaussian copula parameter**, we apply the following model:

$$\rho_t = \Lambda_1\left\{\omega + \beta\rho_{t-1} + a\frac{1}{10}\sum_{j=1}^{10}\Phi^{-1}(u_{1,t-j})\Phi^{-1}(u_{2,t-j})\right\} \quad (7)$$

where $\Lambda_1(x) = (1-e^{-x})(1+e^{-x})^{-1} = \tanh(x/2))$ is the modified logistic transformation, designed to keep ρ_t in $[-1,1]$ at all times.

For the **t-student copula parameter** the model applied is as follow:

$$\rho_t = \Lambda_1\left\{\omega + \beta\rho_{t-1} + a\frac{1}{10}\sum_{j=1}^{10}t_\upsilon^{-1}(u_{1,t-j})t_\upsilon^{-1}(u_{2,t-j})\right\} \quad (8)$$

where $\Lambda_1(x) = (1-e^{-x})(1+e^{-x})^{-1} = \tanh(x/2)$ is the modified logistic transformation, designed to keep ρ_t in $[-1,1]$ at all times.

The **Clayton copula parameter** model is as follow:

$$\alpha_t = \Lambda_2\left\{\omega + \beta\alpha_{t-1} + a\frac{1}{10}\sum_{j=1}^{10}|u_{1,t-j} - u_{2,t-j}|\right\} \quad (9)$$

where $\Lambda_2(x) = e^x$ is the modified logistic transformation to keep in the Clayton parameter domain, $\alpha > 0$.

The **Gumbel copula parameter** model is as follow:

$$\delta_t = \Lambda_3\left\{\omega + \beta\delta_{t-1} + a\frac{1}{10}\sum_{j=1}^{10}|u_{1,t-j} - u_{2,t-j}|\right\} \quad (10)$$

$\Lambda_3(x) = e^x + 1$ to keep Gumbel parameter domain, $\delta > 1$.

The **Fank copula parameter** model is as follow:

$$\theta_t = \omega + \beta\theta_{t-1} + a\frac{1}{10}\sum_{j=1}^{10}|u_{1,t-j} - u_{2,t-j}| \quad (11)$$

where $\theta \in R$.

2.3. Estimation Process

We estimate the copula parameters using the Maximum Likelihood applied to the theoretical joint distribution function. We estimate in a first stage the marginal parameters and in a second stage the copula parameters. This approach is called the inference in margins (IFM) estimation method. Joe (1997) demonstrates that under standard regularity conditions, this two-stage estimation is consistent and the parameters estimated are asymptotically efficient and normal. In the following lines, we describe this method.

Let be $X_1, X_2, \dots X_T$ bidimensional vectors independent and identically distributed *(iid)* $X_i \sim F(x)$ for $i = 1,2, \dots T$. The joint density function is given by

$$f(X_1, X_2, \dots, X_T) = \prod_{i=1}^{T} f(X_{1,i,} X_{2,i} | \alpha, \theta) \quad (12)$$

and the log-likelihood function

$$\ln L(X_1, X_2, \dots, X_T | \alpha, \theta) = \sum_{i=1}^{T} ln\, f(X_{1,i,} X_{2,i} | \alpha, \theta) \quad (13)$$

where $f(X_{1,i,} X_{2,i} | \alpha, \theta)$ is the joint density function of the bidimensional vector X_i and l ln is the natural log. By the Sklar theorem the joint density of this vector can be decomposed into the marginal distributions and a copula density, which is obtained when differentiating equation (2):

$$f(X_{1,i,} X_{2,i} | \alpha, \theta) = c(F_1(X_{1,i}, \alpha_1), F_2(X_{2,i}, \alpha_2); \theta) \prod_{j=1}^{2} f_j(X_{j,i}, \alpha_j) \quad (14)$$

where $f_j(X_{j,i}, \alpha_j)$ is the marginal density function of variable $X_{j,i}$ for $j = 1, 2$; $c(\cdot)$ is the density copula; α_j parameters vector of the marginal density function of variable X_j, and θ is the parameter vector of the copula function. For example, $\theta = [\rho, v]$ for the Student-t copula and $\alpha = [\mu, \omega, \alpha, \beta\ ]$ for the marginals.

We first take log in equation (14)

$$\ln f(X_{1,i,} X_{2,i} | \alpha, \theta) = \ln c(F_1(X_{1,i}, \alpha_1), F_2(X_{2,i}, \alpha_2); \theta) + \sum_{j=1}^{2} \ln f_j(X_{j,i}, \alpha_j) \quad (15)$$

and then substituting in (13)

$$ln\, L(X_1, X_2, \dots, X_T | \alpha, \theta) = \sum_{i=1}^{T} [ln\, c(F_1(X_{1,i}, \alpha_1), F_2(X_{2,i}, \alpha_2); \theta)] + \sum_{i=1}^{T} [ln\, f_1(X_{1,i}, \alpha_1) + ln\, f_2(X_{2,i}, \alpha_2)] \quad (16)$$

The optimal parameter vector is the one that maximizes the previous expression

$$\theta^* = \arg max \sum_{i=1}^{T} [\ln c(F_1(X_{1,i}, \alpha_1), F_2(X_{2,i}, \alpha_2); \theta)] + \sum_{i=1}^{T} [ln\, f_1(X_{1,i}, \alpha_1) + \ln f_2(X_{2,i}, \alpha_2)] \quad (17)$$

As the marginal density of each variable does not depend on the θ, to maximize equation (17) is the same as to maximize the first term

$$\theta^* = \arg max \sum_{i=1}^{T} [\ln c(F_1(X_{1,i}, \alpha_1), F_2(X_{2,i}, \alpha_2); \theta)] \quad (18)$$

Thus, it is possible to maximize the likelihood function in two stages. First, we estimate the parameters of the marginal densities individually, using maximum likelihood

$$\alpha_j{}^* = \arg max \sum_{i=1}^{T} ln\, f_j(X_{j,i}, \alpha_j) \quad (19)$$

Second, the copula parameters can be estimated by resolving the following problem

$$\theta^* = \arg max \sum_{i=1}^{T} ln\, c(u_{1,i}, u_{2,i}; \theta) \quad (20)$$

To go from the first stage to the second stage we must calculate the residual $z_{j,i}$, to which we use the estimated $\hat{\sigma}_{j,i}$ and $\hat{\mu}_{j,i}$:

$$z_{j,i} = \frac{x_{j,i} - \hat{\mu}_{j,i}}{\hat{\sigma}_{j,i}}$$

For $i = 1,2, .., T$. The residuals are then transformed into uniformly distributed variables by inserting them in the univariate distribution of the marginals:

$$u_{j,i} = F_j(z_{j,i}, \alpha_j)$$

2.4. Model Selection criteria

A typical problem that arises when fitting copulas to data is how to decide for the best fitting model. In this study, we use three methods: (i) graphic methods, (ii) sum square error and (iii) information criteria.

First, we follow Genest and Rivest (1993) and compare the empirical copula $(C_T(u))$ with the parametric copulas $(C_\theta(u))$ obtained by Maximum Likelihood Estimation. A scatter plot of these copula distributions (parametric and empirical) should yield a straight line. The empirical copula is given by:

$$C_T(u) = \frac{1}{T} \sum_{i=1}^{T} \mathrm{I}[U_{i,1} \leq u_1, U_{i.2} \leq u_2\ ]$$

$$u = (u_1, u_2) \in [0,1]^2$$

where I(.) is the indicator function taking the value 1 if $U_{i,j} \leq u_j$ and 0 otherwise and $U_i = (U_{i,1}, U_{i,2})$ for $i = 1, \dots n$ are known as pseudo observations which are obtained from

$$U_{i,j} = (T/(T+1)) F_j(x_{i,j}).$$

In the second place, we compare the empirical copula with the parametric copula by calculating the square root of the sum of the differences between them. According to this method, the best copula is the one that minimizes SSE.

$$\text{SSE} = \sqrt{C_T(u) - C_\theta(u)}$$

Another straightforward way to determine which copula provides the best fit to the data is to compare the values of the optimized likelihood function. However the more parameters in the copula, the higher the likelihood tends to be. So to reward parsimony in the copula specification the Akaike information criterion (AIC) or the Bayesian information criterion (BIC) can be applied. The AIC is defined as

$$AIC\ =\ -2\, log(likelihood)\ +\ 2k$$

where k is the number of parameters used in the model.

The value is a measure based on the relative distance between the unknown true likelihood function of the data and the fitted likelihood function of the model. Therefore, a lower AIC means that the model is closer to the truth. The Bayesian information criterion (BIC) is defined as:

$$BIC = -2\,log\,(likelihood) + k \cdot log(T)$$

where T is the number of data and k is the number of parameters used in the model. BIC works similarly as the AIC test, but it penalizes model complexity more heavily. The best fitting model is the one with the lowest BIC[4].

3. Data

For this study we use daily price of IBEX35 index, as a representative of the Spanish stock market, and some international stock markets from different geographic areas. From Europe we have selected FTSE, DAX and CAC40. These indexes are representative of the London stock market, which is the sixth largest by world capitalization, Frankfut stock market which is the tenth largest by world capitalization and French stock market. From Asian we select NIKKEI, HSI, SSE and KOSPI. These indexes are representative of Japan stock market, Hong Kong stock market, China stock market and Korea. The first three markets mentioned rank the third, fourth, fifth by world capitalization. From America we chose S&P500, IPC, IBOVESPA, and MERVAL. These indexes are presentative of the New York stock market, which is the first largest by world capitalization, Mexican stock market, Brazilian stock market and Argentine stocks market[5].

In this study we also include some commodities: Gold, Silver, Copper and Bitcoin. We include the Bitcoin in this group because in the United States it is officially considered as a commodity by decision of the Commodity Futures Trading Commission (CFTC). Table 3 includes a detail description of all of these assets.

[4] For a review of selection criteria of copula model see Fermanian (2005) and Fang et al. (2007) between others.

[5]https://www.ig.com/es/estrategias-de-trading/-cuales-son-las-bolsas-mas-importantes-del-mundo--200703#TSE

Table 3: Description of the data set

Geographical area	Index	Description
US	S&P 500 (^GSPC)	The SP500 measures the stock performance of 500 large companies listed on stock exchanges in the United States (Currency in USD)
Europe	CAC 40 (^FCHI)	The CAC 40 is a benchmark French stock market index. The index represents a capitalization-weighted measure of the 40 most significant stocks among the 100 largest market caps on the Euronext Paris
	DAX (^GDAXI)	The DAX performance index is a blue chip stock market index consisting of the 30 major German companies trading on the Frankfurt Stock Exchange (Currency in EUR)
	FTSE 100 (^FTSE)	is a share index of the 100 companies listed on the London Stock Exchange with the highest market capitalization (Currency in GBP)
Asia	Nikkei 225 (^N225)	The Nikkei 225 (Osaka) is a stock market index for the Tokyo Stock Exchange (Currency in JPY)
	KOSPI composite index (^KS11)	The Korea Composite Stock Price Index or KOSPI is the index of all common stocks traded on the Stock Market Division—previously, Korea Stock Exchange—of the Korea Exchange. It is the representative stock market index of South Korea. (Currency in KRW)
	Hang Seng HSI (^HSI)	The Hang Seng Index is a free float-adjusted market-capitalization-weighted stock-market index in Hong Kong. It is used to record and monitor daily changes of the largest companies of the Hong Kong stock market and is the main indicator of the overall market performance in Hong Kong (Currency in HKD)
	SSE composite index	The SSE Composite Index (Shanghai) also known as SSE Index is a stock market index of all stocks that are traded at the Shanghai Stock Exchange (Currency in CNY)
South America	Merval (^MERV)	The MERVAL Index (Buenos Aires) is the most important index of the Buenos Aires Stock Exchange. It is a price-weighted index, calculated as the market value of a portfolio of stocks selected based on their market share, number of transactions and quotation price (Currency in USD).
	IBOVESPA (^BVSP)	The Ibovespa index (Sao Paolo) is the benchmark index of about 60 stocks that are traded on the B3 (Brasil Bolsa Balcão) (Currency in BRL)
	Mexico IPC (^MXX)	The IPC index seeks to measure the performance of the largest and most liquid stocks listed on the Bolsa Mexicana de Valores (Currency in MXN)
Commodities	GOLD	Commodity Index Gold is designed to track the gold market through futures contracts.
	SILVER	Commodity Index Silver is designed to track the silver market through futures contracts.
	COPPER	Commodity Index Copper is designed to track the copper market through futures contracts.
	BITCOIN	Price of the Bitcoin in dollar ($)

In relation to commodities, Bitcoin has been selected because it is one of the most disruptive financial innovations of the last decade (Feng et al., 2018). The incipient literature on this currency has shown that Bitcoin could act as a risk hedge against traditional assets risks (see Gkillas and Login, 2019; Kang et al., 2019; Klein et al., 2019; Feng et al., 2018; Bouri et al., 2017a; Bouri et al., 2017b; Briere et al., 2015; Eisl et al., 2015 and Dyhrberg, 2016). In this sense, we have found interesting to evaluate the dependency relationship between this asset and the Spanish stock market in order to corroborate whether these results are replicated for the stock Spanish market.

The analysis period run from the January 3, 2000 to June 28, 2019. These data were extracted from Datastream and from the web (Yahoo finance and Investing). The returns are calculated as the log differences in prices multiplied by 100. For the Bitcoin the analysis period goes from August 18, 2011 to Jun 28, 2019.

Figure 1 illustrates the daily price of these assets along the whole period. We observe that the profiles of the evolution of the European indexes, S&P500, NIKKEI and HSI are very similar. They show a strong decline during the first years of the 2000s; thereafter they present an upward trend until the outbreak of the global financial crisis in 2007. From this date, there is a fall in the value of these indexes until the end of 2008 followed by a growth path which has lasted to the end of the analysis period. In the case of IBEX35, after financial crisis increasing and decreasing streaks have alternated, thus showing a behavior oscillating around 10.000 points. The rest of indexes considered show a behavior somewhat different. Some of them show an upward trend along the whole period although with some important interruptions as the observed between 2007-2008 (MERVAL, IPC, KOSPI). The commodities and Shanghai markets show also different behavior, especially Bitcoin. Gold and Silver grow almost steadily until 2010. This growth was especially intense during the financial crisis, which is consistent with the fact that these assets are safe haven. Since then, the prices of these assets have tended to fall. As the stock markets, the price of the Copper was strongly affected by the financial crisis. To last, Bitcoin price shows a spectacular growth going from $11 in august, 2011 to around $12.300 currently. This growth was especially strong between 2014 and 2017, which was almost exponential. Since then it has shown a down trend although in recent months it has come back to go up.

Figure 2 illustrates the returns of these assets along the whole period. We observe that the range of the fluctuation of the returns change over time and these variations evolve according to the idea of cluster in volatility (Mandelbrot, (1963)). Note that the range of fluctuations of the Bitcoin is much higher than the rest of assets considered.

Table 4 displays the descriptive statistics of the returns. The average return is positive for all assets except for IBEX35 and CAC40 indexes. Remark that the European indexes provide the lowest average return while commodities and American assets, except S&P500, provide the highest average return. The standard deviation of these assets is also the highest but, in most cases, the high return offsets the excess of risk. The asset that provides highest Sharpe ratio is Bitcoin followed by MERVAL, IPC, Gold and IBOVESPA. European indexes perform the worst. The return distribution of all assets considered except Gold, is negatively skewed and exhibits an important excess kurtosis suggesting leptokurtic behavior. In all cases, the value of the Jarque-Bera statistic indicates the departure from normality.

Table 4: Descriptive Statistics of some international stock indices market and four commodities daily returns for Jan 03, 2000 to Jun 28, 2019 (*)

	Mean	Std. Dev.	Skew	Kurtosis	Jarque-Bera	ARCH	Correlation
Panel (a) European stock indices market							
Spain	-0.004	1.45	-0.08	6.12	7706.7*	33.2*	
France	-0.001	1.41	-0.04	5.20	5607.7*	447.4*	0.87
United Kingdom	0.002	1.16	-0.15	6.46	8558.4*	572.6*	0.77
Germany	0.012	1.46	-0.05	4.61	4373.4*	249.8*	0.79
Panel (b) American stock indices market							
United States	0.014	1.20	-0.22	8.59	15054.0*	886.6*	0.51
Argentina	0.090	2.15	-0.15	4.00	3175.3*	535.0*	0.36
Brazil	0.037	1.76	-0.09	3.80	2895.4*	1039*	0.42
Mexico	0.037	1.27	-0.01	5.46	6047.5*	234.4*	0.46
Panel (c) Asian stock indices market							
Japan	0.002	1.50	-0.41	6.37	8201.2*	1140.7*	0.27
Hong Kong	0-010	1.47	-0.11	8.07	13017.0*	1201.9*	0.34
Korea	0.014	1.49	-0.60	7.02	10121.0*	190.2*	0.27
Shanghai	0.015	1.55	-0.35	5.07	5333.1*	99.1*	0.09
Panel (d) Commodities							
Gold	0.032	1.34	0.04	14.49	43330.0*	1032.7*	0.03
Silver	0.019	1.87	-0.86	8.30	16314*	26,4*	0.05
Copper	0.024	1.75	-0.18	4.13	3500*	287.3*	0.18
Bitcoin	0.340	5.90	-0.96	19.89	34220*	45.5*	0.01

The analysis period for Bitcoin cover from Aug 18, 2011 to Jun 28, 2019. Std. Dev. is the standard deviation. ARCH is the test for autoregressive conditional heteroscedasticity by Engle (1982) at the 10 lag. Correlation is the Pearson correlation coefficient between IBEX35 and the rest of indices considered. (*) Denotes significance at 1% level.

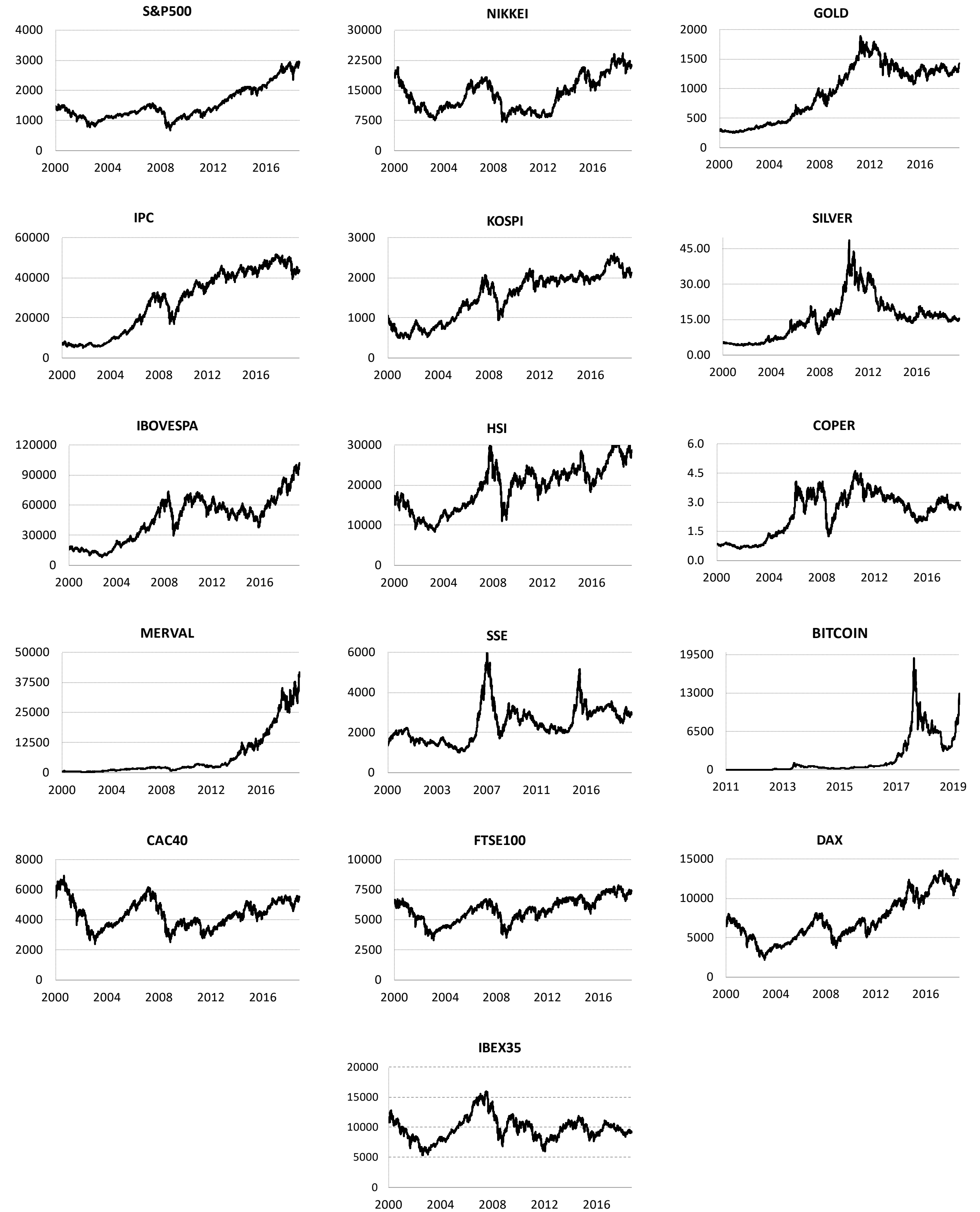

Figure 1 illustrates some international stock indices market from America (S&P500, MERVAL, IBOVESPA and IPC) Europe (IBEX35, CAC40, FTSE100 and DAX) Asian (NIKKEI, HSI, KOSPI and SSE) and four commodities (Gold, Silver, Copper and Bitcoin).

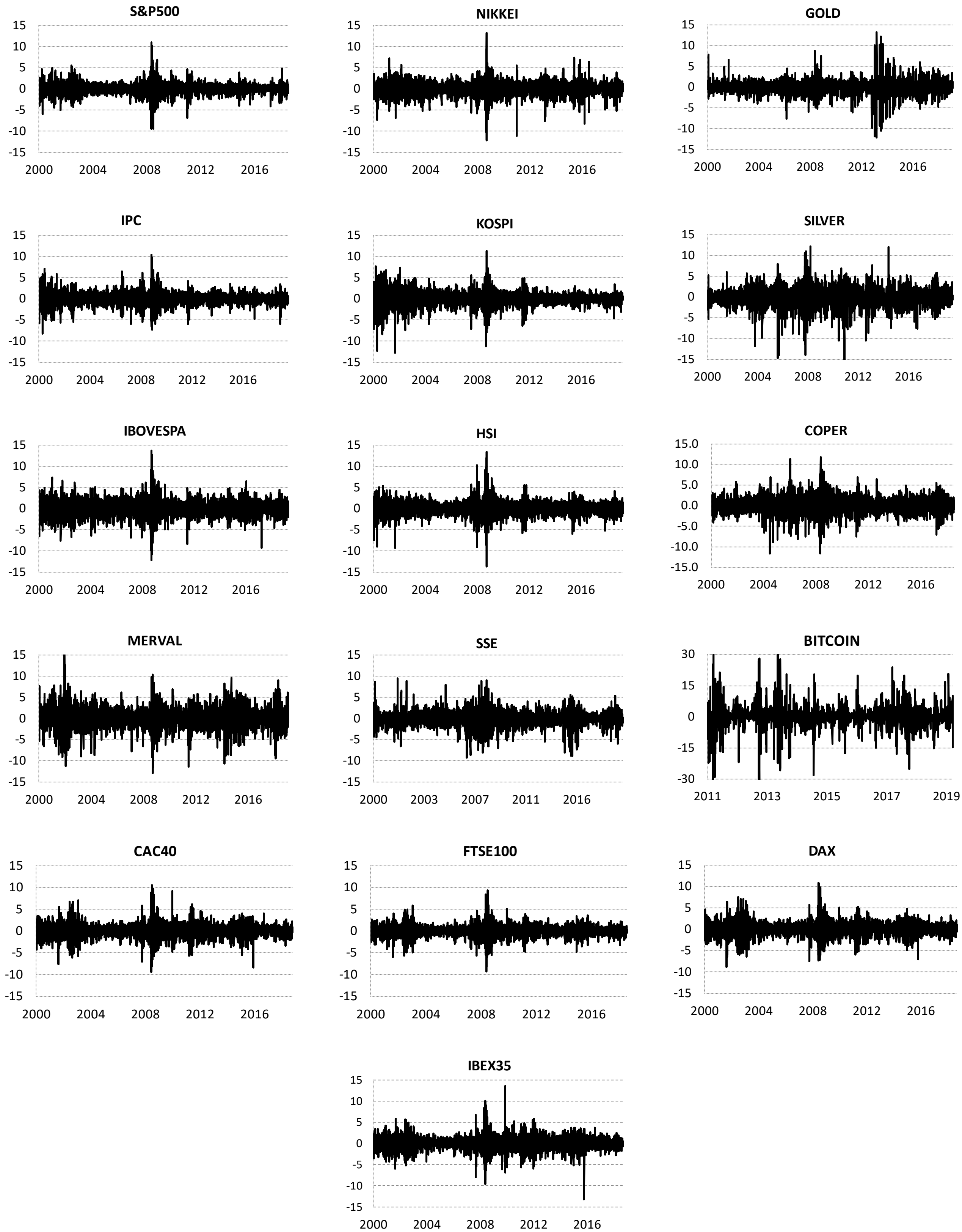

Figure 2 illustrates some returns of international stock indices market from America (S&P500, MERVAL, IBOVESPA and IPC) Europe (IBEX35, CAC40, FTSE100 and DAX) Asian (NIKKEI, HSI, KOSPI and SSE) and four commodities (Gold, Silver, Copper and Bitcoin).

The value of the ARCH statistic for conditional heteroscedasticity confirms that there exists ARCH effect in the return of the IBEX35 and rest of the assets, which justifies the appropriateness of using a GARCH framework to model the conditional volatility.

Table 4 also reports the Pearson correlation coefficient. We deduct from this coefficient that the European indexes offer limited diversification possibilities as they reach values over 0.75 in all cases. The diversification possibilities via American assets allocation are also limited but they are higher than these offered by the European countries. In the ranking, Asian countries are the bests, especially China that keeps a null correlation with Spanish stock market. Regarding the commodities, Bitcoin, Gold and Silver keep on a null correlation meanwhile Copper presents a positive correlation but much lower than the observed for the stock indexes.

4. Empirical results

4.1. Modeling marginals

The first step in modelling marginal consists in assuming distribution for the returns. To assess the distribution that fit best the data we compare several fat tail distributions:

i) Student-t,
ii) Generalized error distribution (GED),
iii) Skew Student-t and skew GED.

Normal distribution is excluded of the comparison because, as we show in the previous section, our data set shows strong evidence against the normality hypothesis (see Table 4). For selecting the distribution that fit best we focus on log Likelihood and two information criteria: AIC and BIC (see Table 5). The greater the value of the likelihood function, the better the setting. Regard to the information criteria, the lower these are, the better the adjustment will be.

According to these criteria, Student-t distribution is the best in fitting data for S&P500(US), DAX(German), KOSPI(Korea), SSE(Chinese), Gold and Silver. For the rest of the assets the best distribution is the generalized error distribution (GED). Figure 3 illustrate the QQ-plot of the distribution fitted: Student-t and GED. A good fit is observed in all cases.

Table 5: Likelihood, Akaike information criterium (AIC) and BIC Ccriterium
Intenational stocks markets and commodities

		std	sstd	GED	sGED
IBEX35	Likelihood AIC BIC	8488.8 -1.86 3.23	8482.6 0.14 6.92	**8495.6** **-1.86** **3.22**	8484.9 0.14 6.92
FTSE	Likelihood AIC BIC	7233.56 -1.719 3.364	7229.02 0.282 7.059	**7256.43** **-1.721** **3.362**	7250.18 0.279 7.057
DAX	Likelihood AIC BIC	**8507.0** **-1.860** **3.223**	8499.472 0.141 6.919	8490.27 -1.858 3.225	8479.5 0.143 6.921
CAC40	Likelihood AIC BIC	8336.35 -1.842 3.241	8330.67 0.159 6.936	**8350.31** **-1.843** **3.240**	8346.35 0.157 6.934
SP500	Likelihood AIC BIC	**7177.9** **-1.712** **3.371**	7170.26 0.289 7.066	7159.4 -1.710 3.373	7154.4 0.291 7.068
IBOVESPA	Likelihood AIC BIC	9360.9 -1.943 3.140	9358.5 0.058 6.835	**9385.5** **-1.945** **3.138**	9383.2 0.055 6.833
MERVAL	Likelihood AIC BIC	10048 -2.0042 3.0789	10046.2 -0.0040 6.7734	**10049.9** **-2.0043** **3.0787**	10048.2 -0.004 6.773
IPC	Likelihood AIC BIC	7654.7 -1.768 3.315	7648.4 0.233 7.010	**7683.4** **-1.771** **3.312**	7679.7 0.229 7.007
NIKKEI	Likelihood AIC BIC	7652.7 -1.768 3.315	7648.4 0.233 7.010	**7683.4** **-1.771** **3.312**	7679.7 0.229 7.007
HSI	Likelihood AIC BIC	8179.9 -1.825 3.258	8176.4 0.175 6.952	**8182** **-1.826** **3.257**	8177.8 0.175 6.952
SSE	Likelihood AIC BIC	**8552.2** **-1.864** **3.219**	8551 0.136 6.913	8496.8 -1.859 3.225	8496.8 0.141 6.919
KOSPI	Likelihood AIC BIC	**8134.5** **-1.821** **3.262**	8126.5 0.180 6.958	8117.2 -1.819 3.264	8113.6 0.182 6.959
Bitcoin	Likelihood AIC BIC	**5954.9** **-1.550** **3.533**	5954.8 0.450 7.228	5917.6 -1.544 3.539	5915.8 0.456 7.233
Gold	Likelihood AIC BIC	7630.4 -1.765 3.318	7630.4 0.235 7.012	**7668.4** **-1.769** **3.314**	7669.5 0.230 7.008
Silver	Likelihood AIC BIC	**10493.1** **-2.042** **3.041**	10487.4 -0.041 6.736	10486.1 -2.041 3.042	10477.3 -0.040 6.737
Cooper	Likelihood AIC BIC	**9346.1** **-1.941** **3.142**	9346.0 0.059 6.836	9342.7 -1.941 3.142	9342.3 0.059 6.836

Note: std, denotes the Student-t distribution; sstd, denotes de skewed Student-t distribution; GED denotes the generalize error distribution; SGED denotes the skewed generalize error distribution. We highlight in bold the distribution that provides the best fi

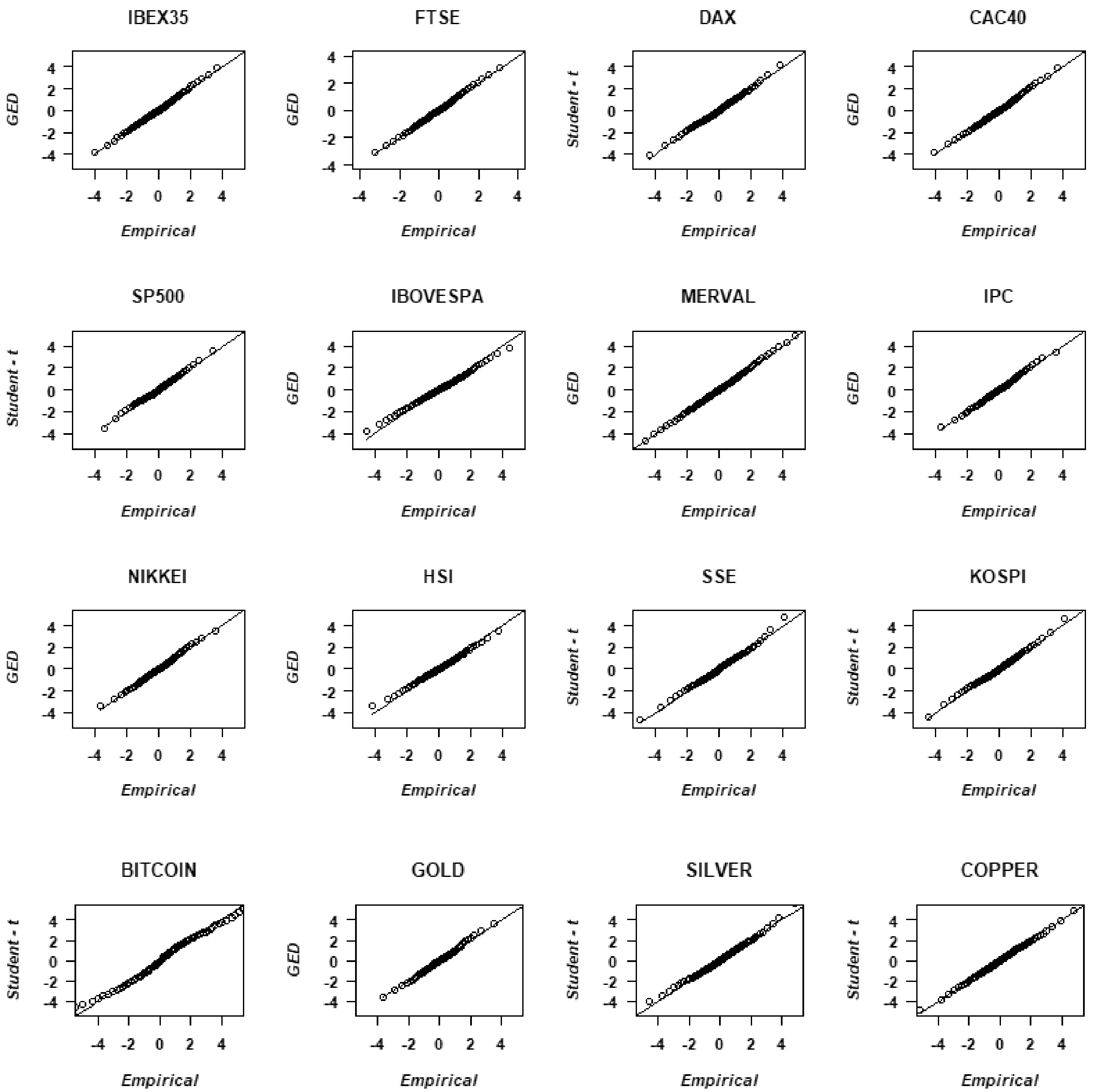

Figure 3 reports the QQ-plot of the distribution fitted, which are: Student-t (IBEX35, DAX, SP500, SSE, KOSPI, Bitcoin, Silver and Copper) and GED (FTSE, CAC40, IPC, IBOVESPA, MERVAL, NIKKEI, HIS and Gold)

4.2. Modeling the conditional variance

In this section, we estimate jointly an autoregressive model AR(p) for the conditional returns and an APARCH model for the conditional variance. This last model captures some characteristics of financial returns as well as memory and asymmetric effects.

We estimate the parameters of the AR(p)-APARCH model with a Student-t distribution with v degrees of freedom for S&P500, DAX, KOSPI, SSE, Gold and Silver. For the rest of the assets we assume a GED distribution. The results of the estimations are reported in Table 6.

Overall, we observe that the estimations for the stocks indexes are quite similar. We find that the persistence of the volatility, measured by the parameter β is high being around 0.93; just only MERVAL, NIKKEI and Bitcoin show a persistence somewhat lower. The parameter γ which captures the *leverage effect* is positive and statistically significant, indicating that volatility tends to be higher after negative returns. Remark also that S&P500 and European indexes provide higher *leverage effect* than the rest of indexes considered. On the other hand, for all commodities except Copper, parameter γ is negative and statistically significant which means that volatility tends to be higher after positive returns. This is known as an *inverse leverage effect* a prominent feature of commodities such as Gold (Baur, 2012) [6].

The power parameter δ is around 1 for S&P500, European indexes and Asian indexes which means that modeling the standard deviation seems better rather than the variance. However, for the rest of the assets δ parameter takes a value somewhat higher moving around 1.117 and 1.829. In all cases, the ARCH test reveals that the APARCH model captures adequately the volatility of the innovations.

Table 6: Estimation results for APARCH model

	μ	$\emptyset$	ω	α	γ	β	δ	shape	ARCH
Panel (a) European stock indices market									
IBEX35	0.020	--	0.021*	0.060*	0.961*	0.934*	1.113*	1.530*	0.15
CAC40	0.010	-0.031*	0.022*	0.073*	1.000*	0.926*	0.931*	1.573*	0.71
FTSE100	0.004	-0.025**	0.019*	0.070*	1.000*	0.923*	1.028*	1.650*	0.71
DAX	0.033*	--	0.023*	0.068*	1.000*	0.924*	1.078*	10.000*	1.87
Panel (b) American stock indices market									
S&P500	0.041*	-0.053*	0.019*	0.085*	1.000*	0.915*	0.972*	7.191*	1.90
MERVAL	0.109*	0.044*	0.133*	0.104*	0.229*	0.863*	1.829*	1.311*	3.39
IBOVESPA	0.039**	--	0.044*	0.057*	0.567*	0.926*	1.354*	1.616*	0.22
IPC	0.043*	--	0.017*	0.077*	0.505*	0.919*	1.372*	1.343*	0.91
Panel (c) Asian stock indices market									
NIKEEI	0.023	--	0.045*	0.101*	0.613*	0.887*	0.991*	1.464*	1.16
HSI	0.038*	--	0.017*	0.059*	0.535*	0.938*	1.110*	1.429*	6.17
KOSPI	0.046*	--	0.011*	0.076*	0.481*	0.931*	1.107*	6.568*	0.61
SSE	0.038**	--	0.011*	0.078*	0.145*	0.936*	0.138*	4.315*	1.38
Panel (d) Commodities									
Gold	0.048*	-0.068*	0.011*	0.044*	-0.249*	0.960*	0.618*	3.134*	193*
Silver	0.059*	-0.062*	0.010*	0.050*	-0.294*	0.960*	1.227*	3.723*	13.9
Copper	0.013	-0.084*	0.014*	0.049*	0.078	0.951*	1.600*	1.302*	0.50
Bitcoin	0.286*	--	0.183*	0.348*	-0.165*	0.837*	1.117*	2.312*	0.02

ARCH is the test for autoregressive conditional heteroskedasticity by Engle (1982) at the 10 lags. (**) and (*) denote significance at 5% and 1% level respectively.

Figure 4 show the volatility estimated. For all the assets considered, we observe an important increase of volatility during the global financial crisis 2008-2009. Many of them also exhibit a high volatility the first years of the 2000s decade, after the dotcom bubble[7]. To last, remark that the Gold and Silver along with the European indexes exhibit lowest peaks of volatility while Bitcoin exhibits the highest followed by far by MERVAL, IPC, NIKKEI and HSI indexes.

[6] Baur (2012) studies the volatility of Gold and demonstrates that there is an inverted asymmetric reaction to positive and negative shocks, i.e. positive shocks increase the volatility by more than negative shocks. The paper argues that this effect is related to the safe-haven property of gold. Although Bitcoin is a currency, it exhibits also an *inverse leverage effect*. This result is in line with other studies (see; Klein et al., 2018 and Bouri et al., 2017b)

[7] The dotcom bubble, also known as the internet bubble, was a rapid rise in U.S. technology stock equity valuations fueled by investments in internet-based companies during the bull market in the late 1990s. During the dotcom bubble, the value of equity markets grew exponentially, with the technology-dominated Nasdaq index rising from under 1000 to more than 5000 between the years 1995 and 2000. In 2001 and through 2002 the bubble burst, with equities entering a bear market.

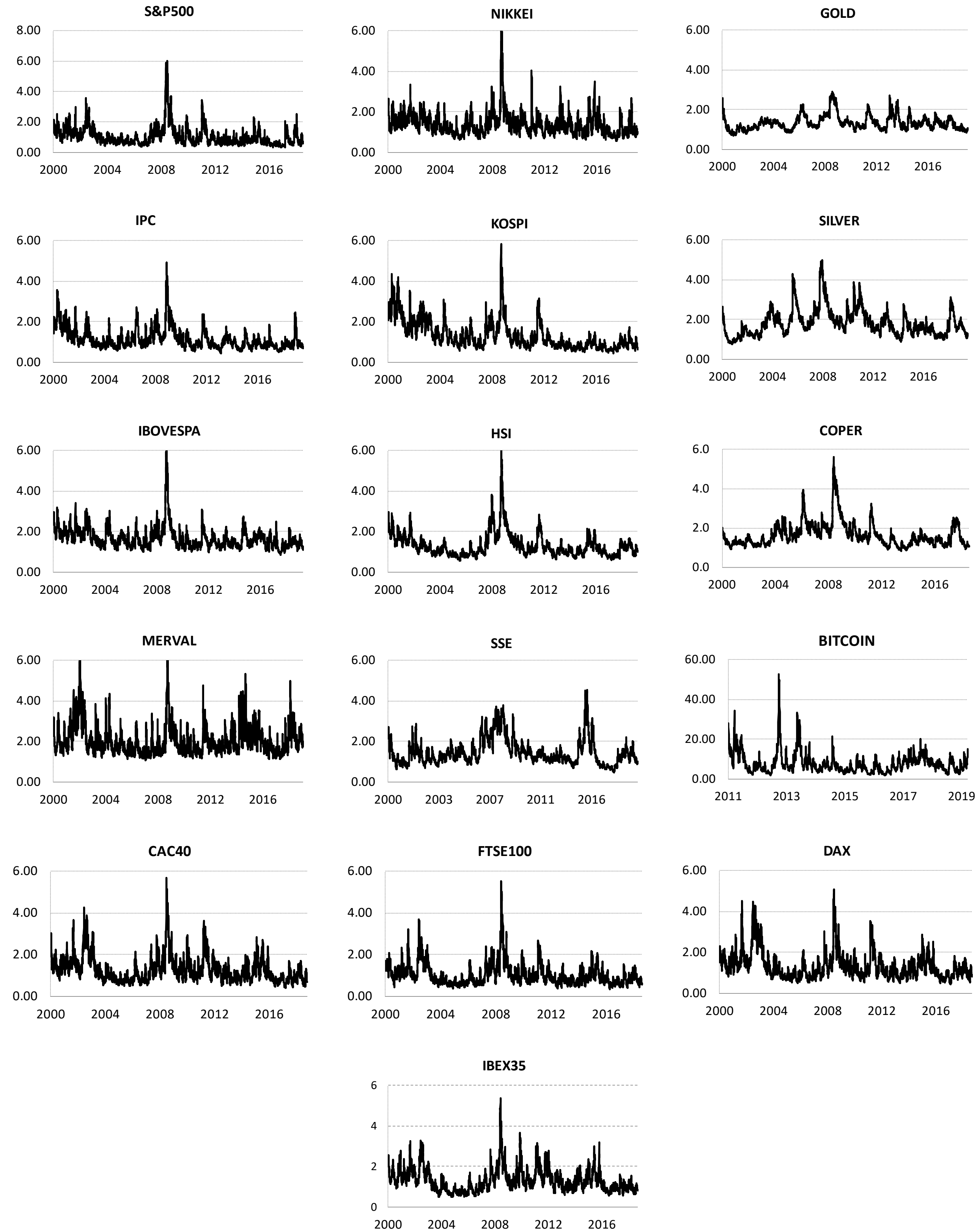

Figure 4 illustrates the conditional standard deviation estimated through the APARCH model.

4.3. Modeling the dependence structure using copulas

Table 7 reports copula estimation for dependence study between the Spanish stock market and the European stock markets. In this table we also include the Kendall's τ tau derived from the copula functions[8]; this rank correlation measure is very useful, because it allows comparative analyses of global dependence structures when copulas are different and, consequently, the copulas dependence parameters are non-comparable. This measure always varies between −1 and 1, and is invariant to non-linear transformations, as long as it is monotonic, as it is the case of probability integral transforms of marginal variables in the context of the copula theory (Horta et al. 2010). Taking this into account, in this study, we use the Kendall's τ to assess global dependence structures between markets.

The dependence structure between the Spanish stock market and the European stock markets is very high although some differences are observed between markets. According to the Kendall's τ the highest dependence is observed with the French market which moves between 0.536 and 0.666 depending on the copula. The lowest dependence is observed with the UK market which is around 0.520. The dependence with the German market is around 0.55, although varies depending on the copula. These results indicate that European markets offer limitated diversification possibilities for the Spanish investors and vice versa.

We are also interested in finding the copula model that fits best the dependence structure between all these markets. Therefore, we include in Table 7 the Akaike information criterion (AIC), the Bayesian information criterion (BIC) and the square root of the sum errors (SSE) obtained from comparing the empirical copula and the theoretical copula. We obtain the smallest values of AIC, BIC and the sum square errors for Student-t copula. Just only in the case of DAX the sum square errors point the Frank copula as the best in fitting data. A visual inspection of the scatter plot of the data is also included (Figure 5). These plots, suggest that the fitting is quite good for all copula models, especially for the elliptical copulas.

Table 7: Studying structure dependence between the Spanish stock market (IBEX35) and some European stock markets.

Copula	Copula Parameter		**Kendall's τ**	**AIC**	**BIC**	**SSE**	**Lower tail**	**Upper tail**
Panel (a) Spain (IBEX35) – France (CAC40)								
Gaussian (ρ)	0.848*		0.644	21629.9	4.378	0.184	0	0
Student-t (ρ, v)	0.853*		0.650	**21472.3**	**4.346**	**0.164**	0.464	0.464
Clayton (α)	2.315*	v = 7*	0.536	22808.9	4.616	0.701	0.741	0
Gumbel (δ)	2.809*		0.644	21715.4	4.395	0.527	0	0.720
Frank	10.12*		0.666	21937.6	4.440	0.572	0	0
Panel (b) Spain (IBEX35) – UK (FTSE)								
Gaussian (ρ)	0.729*		0.520	23855.1	4.895	0.270	0	0
Student-t (ρ, v)	0.738*		0.528	**23734.8**	**4.871**	**0.232**	0.301	0.301
Clayton (α)	1.384*	v = 7*	0.409	24642.0	5.057	1.852	0.606	0
Gumbel (δ)	2.104*		0.525	23863.3	4.897	0.658	0	0.610
Frank	6.619*		0.541	23985.4	4.922	0.635	0	0
Panel (c) Spain (IBEX35) – Germany (DAX)								
Gaussian (ρ)	0.795*		0.585	22823.0	4.637	0.915	0	0
Student-t (ρ, v)	0.803*		0.593	**22665.6**	**4.605**	0.863	0.409	0.409
Clayton (α)	1.843*	v = 6*	0.479	23724.3	4.820	2.485	0.686	0
Gumbel (δ)	2.424*		0.587	22902.8	4.653	1.081	0	0.668
Frank	8.264*		0.609	23026.7	4.678	**0.585**	0	0

AIC denotes the Akaike information criterion; BIC denotes the Bayesian information criterion; SSE denotes the root of the sum square error. In bold we remark the lowest values of BIC, AIC and SSE. (*) denotes significance at 1% level.

[8] The dependence structure between variables may be characterized by a copula, but may also be expressed using scalar synthetic measures derived from the same copula. An example of such measures are rank correlation coefficients, as the Kendall's τ or the Spearman's ρ.

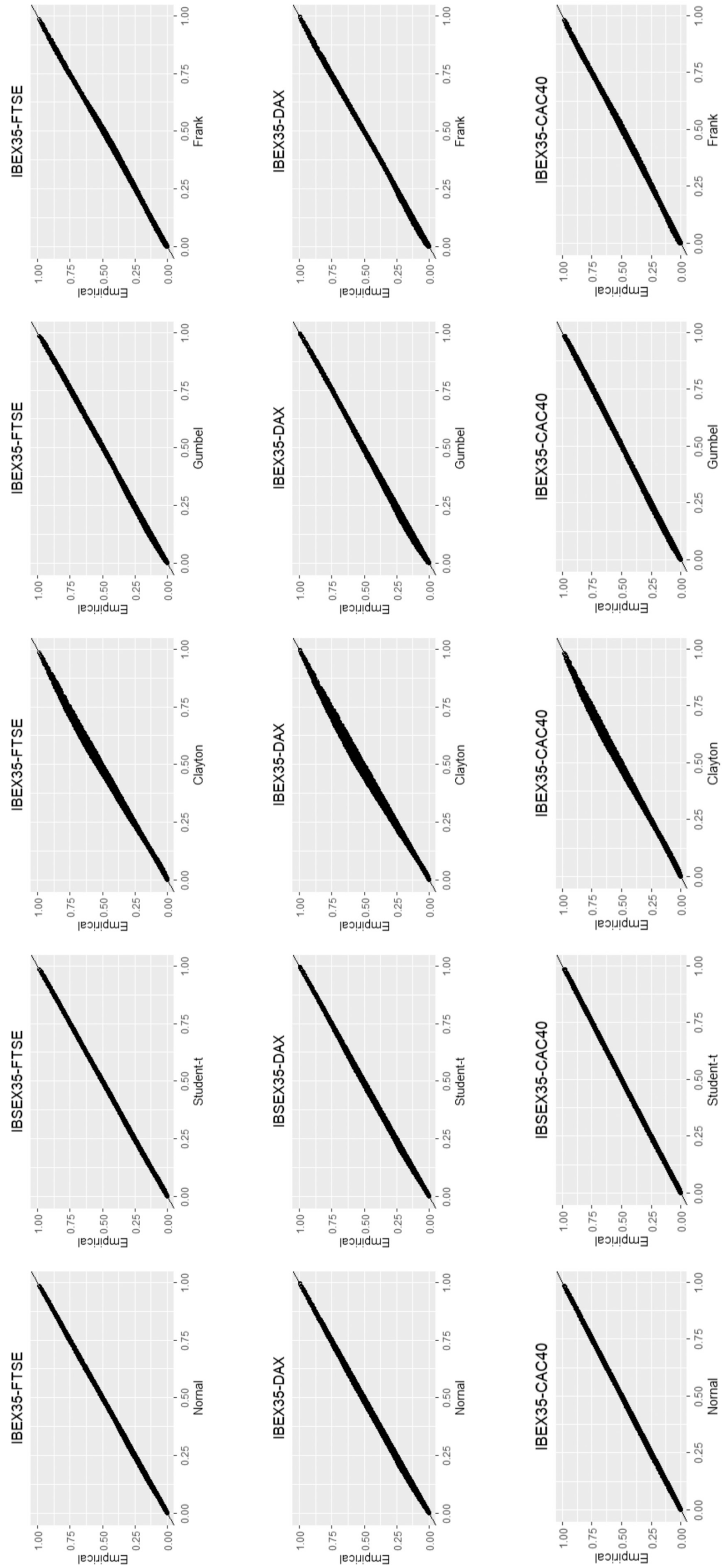

Figure 5: Scatter plots between the empirical copula and the parametric copula.

European markets

Unlike the traditional methods used to measure dependence between markets, the copula function allows us to know the dependence in tails distribution. If there is no tail dependence among the returns in a portfolio then there is little risk of simultaneous very negative/positive returns and therefore probability of occurrence of an extreme negative/positive return on the portfolio will be low. However, if there exist tail dependence, then the probability of occurrence of extreme negative/positive returns simultaneously can be high. Hence, it is important to consider tail dependence when assessing the diversification benefit and risk of a portfolio (Rajwani et al. 2019).

The Student-t copula has symmetric tail dependence regardless of whether the markets are booming or crashing. According to the student-t copula, which is the best in fitting data, the tail dependence is 0.462 for the French market, 0.301 for the UK market and 0.409 for the German market. These data indicate that the probability of joint market crashes in these countries is very high so that the portfolio managers should become more alert and take account of this comovement.

Table 8 reports copula estimation for dependence study between the Spanish stock market and the American stock markets. According to the Kendall's tau these markets offer higher diversification possibilities than the Europeans. Although there are some differences between copulas, overall, the dependence with the Americans markets is around 0.34 with USA market, 0.29 with Mexico stock market, 0.24 with Argentina market and 0.22 with the Brazil market.

A visual inspection of the scatter plot of the data suggests that the fitting is quite good for all copula models, not observing differences between them (Figure 6). According to the AIC and BIC criteria, the student-t copula is the best in fitting data for S&P500, MERVAL and IBOVESPA indexes while the sum square errors points the normal copula as the best. Just only for IPC all criteria match pointing the normal as the best copula model.

Table 8: Studying structure dependence between the Spanish stock market (IBEX35) and some American stock markets.

Copula	Copula Parameter		**Kendall's τ**	**AIC**	**BIC**	**SSE**	**Lower tail**	**Upper tail**
				Panel (a) IBEX35 - S&P500				
Gaussian (ρ)	0.514*		0.343	27196.9	5.643	**0.327**	0	0
Student-t (ρ, v)	0.514*		0.343	**27150.4**	**5.633**	0.328	0.085	0.085
Clayton (α)	0.657*	v = 10*	0.247	27528.3	5.711	1.428	0.348	0
Gumbel (δ)	1.530*		0.346	27211.1	5.645	0.736	0	0.427
Frank	3.503*		0.345	27340.8	5.673	0.669	0	0
				Panel (b) IBEX35 - MERVAL				
Gaussian (ρ)	0.345*		0.224	25858.0	5.490	**0.300**	0	0
Student-t (ρ, v)	0.344*		0.224	**25829.9**	**5.484**	0.335	0.020	0.020
Clayton (α)	0.395*	v = 13*	0.165	25943.1	5.508	0.922	0.173	0
Gumbel (δ)	1.278*		0.217	25923.6	5.504	0.677	0	0.280
Frank	2.118*		0.219	25922.6	5.503	0.444	0	0
				Panel (c) IBEX35 - IBOVESPA				
Gaussian (ρ)	0.377*		0.246	26226.0	5.497	**0.415**	0	0
Student-t (ρ, v)	0.376*		0.245	**26206.8**	**5.493**	0.446	0.015	0.015
Clayton (α)	0.448*	v = 15*	0.183	26330.6	5.519	0.872	0.213	0
Gumbel (δ)	1.311*		0.237	26311.8	5.515	0.806	0	0.302
Frank	2.324*		0.239	26314.6	5.515	0.578	0	0
				Panel (d) IBEX35 - IPC				
Gaussian (ρ)	0.443*		0.291	**26066.0**	**5.409**	**0.272**	0	0
Student-t (ρ, v)	0.444*		0.292	26104.0	5.417	0.313	0.019	0.0191
Clayton (α)	0.539*	v = 16*	0.212	26347.6	5.467	1.261	0.276	0
Gumbel (δ)	1.404*		0.287	26206.6	5.438	0.711	0	0.361
Frank	2.876*		0.294	26223.0	5.441	0.495	0	0

AIC denotes the Akaike information criterion; BIC denotes the Bayesian information criterion; SSE denotes the root of the sum square error. In bold we remark the lowest values of BIC, AIC and SSE. (*) denotes significance at 1% level.

According to the Student-t copula, which is the best in fitting data according to the AIC and BIC criteria, tail dependence between the Spanish stock market and the American stocks markets is very small, being close to zero for all countries. This result is very important as it suggests that the dependence in crisis periods, when it is more necessary that diversification works, is much reduced

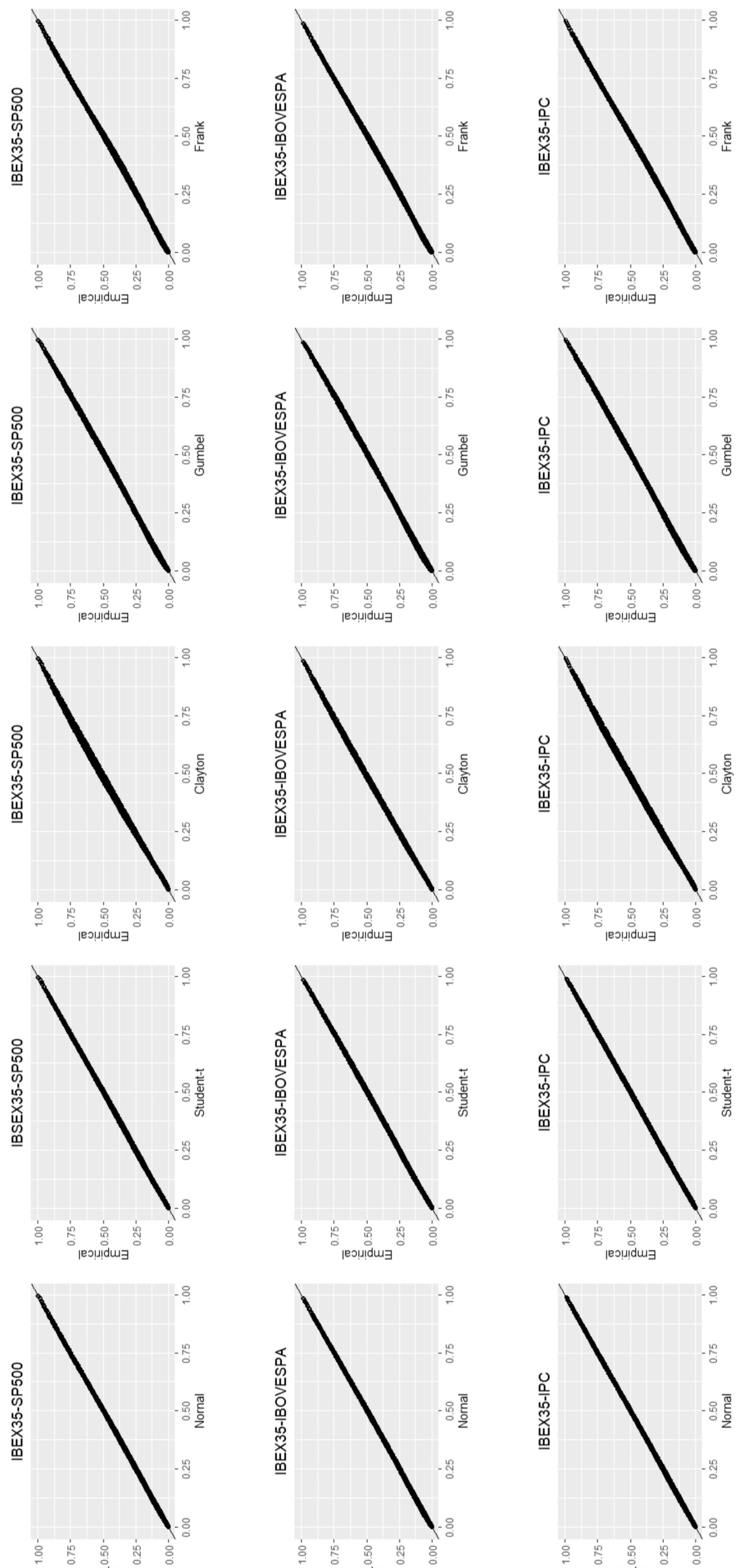

Figure 6: Scatter plots between the empirical copula and the parametric copula. **American markets**

Copula estimations for dependence study between the Spanish stock market and the Asian stock markets are reported in Table 9. Overall, we observe that regardless of copula used, the dependence between the Spanish market and the Asian markets is very low although some differences among countries are found. The highest level of dependence is found for Hong Kong market which moves around 0.19 in most cases, follows by Korea (0.16) and Japan (0.15). These countries may offer high diversification possibilities for the Spanish investors moreover when we observed that the low tail dependence is even more reduced being lower than 0.001 in all cases. The lowest dependence is found for China market where the dependence is around 0.05 being 0 in the low tail. These results indicate that assets negotiated in the Chinese (Shanghai) market may be hedge assets rather than diversifier assets. Here we adopt the definition of hedge asset given by Baur and Lucey (2010). These authors define a hedge as an asset that has a negative dependence or null with another asset or portfolio on average. A diversifier is defined as an asset that has a positive dependence structure (but not perfectly) with another asset in a portfolio.

Table 9: Studying structure dependence between Spanish stock market (IBEX35) and some Asian stock markets.

Copula	Copula Parameter	**Kendall's τ**	**AIC**	**BIC**	**SSE**	**Lower tail**	**Upper tail**
			Panel (a) IBEX35 - NIKKEI				
Gaussian (ρ)	0.259*	0.167	25976.7	5.565	**0.349**	0	0
Student-t (ρ, υ)	0.257*	0.165	**25972.6**	**5.564**	0.506	$6.9\ 10^{-06}$	$6.9\ 10^{-06}$
Clayton (α)	0.261* υ = 43*	0.115	26024.0	5.575	0.703	0.069	0
Gumbel (δ)	1.183*	0.154	26048.2	5.580	0.681	0	0.203
Frank	1.483*	0.157	26024.9	5.575	0.395	0	0
			Panel (b) IBEX35 - HSI				
Gaussian (ρ)	0.315*	0.204	26340.2	5.526	**0.416**	0	0
Student-t (ρ, υ)	0.314*	0.203	**26331.7**	**5.525**	0.482	0.001	0.001
Clayton (α)	0.343* υ = 25*	0.146	26432.6	5.546	0.798	0.132	0
Gumbel (δ)	1.239*	0.193	26402.2	5.539	0.760	0	0.2505
Frank	1.882*	0.196	26400.0	5.539	0.542	0	0
			Panel (c) IBEX35 - KOSPI				
Gaussian (ρ)	0.267*	0.172	27864.4	5.905	**0.321**	0	0
Student-t (ρ, υ)	0.265*	0.171	**27860.7**	**5.904**	0.458	$5.14\ 10^{-05}$	$5.14\ 10^{-05}$
Clayton (α)	0.267* υ = 35*	0.118	27926.5	5.918	0.797	0.074	0
Gumbel (δ)	1.196*	0.164	27921.7	5.917	0.669	0	0.214
Frank	1.587*	0.168	27898.7	5.912	0.367	0	0
			Panel (d) IBEX35 - SSE				
Gaussian (ρ)	0.085*	0.054	29041.9	6.141	0.233	0	0
Student-t (ρ, υ)	0.084*	0.053	29034.4	6.140	0.413	$2.54\ 10^{-05}$	$2.54\ 10^{-05}$
Clayton (α)	0.092* υ = 28*	0.044	**29029.2**	**6.139**	**0.215**	$0.5\ 10^{-03}$	0
Gumbel (δ)	1.042*	0.040	29057.6	6.144	0.356	0	0.055
Frank	0.475*	0.052	29046.9	6.142	0.241	0	0

AIC denotes the Akaike information criterion; BIC denotes the Bayesian information criterion; SSE denotes the root of the sum square errors. In bold we remark the lowest values of BIC, AIC and SSE. (*) denotes significance at 1% level.

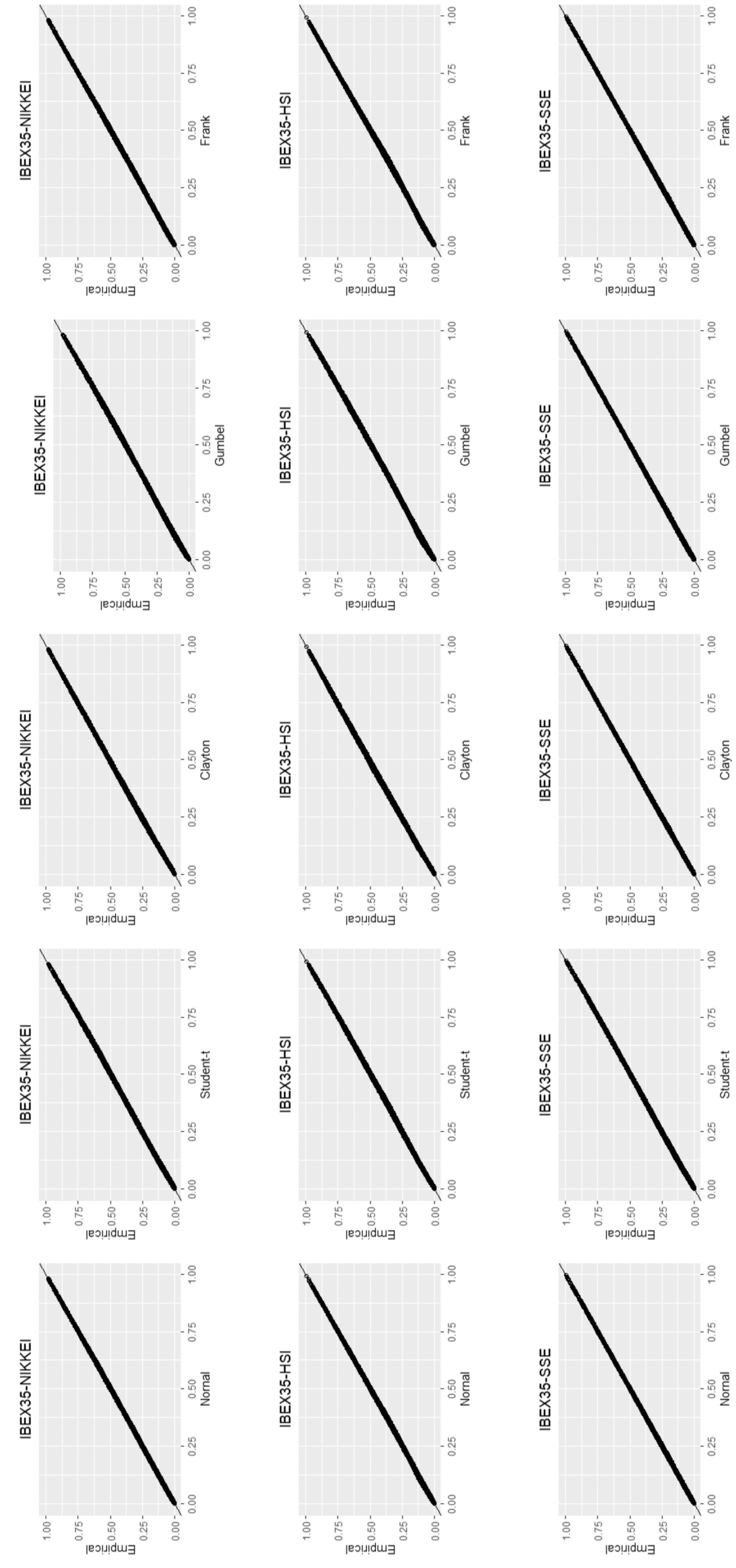

Figure 7: Scatter plots between the empirical copula and the parametric copula. **Asian markets**

These results suggest that there is significant potential for risk diversification by investing in the Shanghai market by Spanish investors. This result has not been documented in the existing literature and provides a new insight into risk diversification.

To last, in Table 10 we report the copula estimations for the study of dependence between the Spanish stock market and the commodities markets. We observe that Gold is clearly a hedge asset as overall it has negative dependence with the Spanish market. Bitcoin may also act as a hedge asset as it keeps a null dependence with the Spanish stock market[9]. Copper shows the highest dependence being around 0.16 meanwhile Silver has a much reduced dependence (around 0.05).

Table 10: Studying structure dependence between Spanish stock market (IBEX35) and some commodities.

Copula	Copula Parameter		**Kendall's τ**	**AIC**	**BIC**	**SSE**	**Lower tail**	**Upper tail**
				Panel (a) IBEX35 - Gold				
Gaussian (ρ)	-0.029*		-0.018	26096.7	5.401	0.387	0	0
Student-t (ρ, v)	-0.026**		-0.017	**26078.7**	**5.397**	0.440	$1.13\ 10^{-4}$	$1.13\ 10^{-4}$
Clayton (α)	0.009**	v = 20*	0.004	26100.4	5.402	0.431	$3.04\ 10^{-34}$	0
Gumbel (δ)	1.003**		0.009	26112.5	5.397	0.430	0	$2.1\ 10^{-8}$
Frank	-0.158*		-0.017	26097.9	5.401	**0.332**	0	0
				Panel (b) IBEX35 - Silver				
Gaussian (ρ)	0.082*		0.052	29941.4	6.156	0.369	0	0
Student-t (ρ, v)	0.082*		0.052	**29930.4**	**6.153**	0.463	$1.6\ 10^{-4}$	$1.6\ 10^{-4}$
Clayton (α)	0.092*	v = 22*	0.044	29930.8	6.154	**0.263**	$5.5\ 10^{-4}$	0
Gumbel (δ)	1.046*		0.044	29948.0	6.157	0.473	0	0.060
Frank	0.463*		0.051	29945.2	6.157	0.373	0	0
				Panel (c) IBEX35 - Copper				
Gaussian (ρ)	0.266*		0.171	28566.9	5.929	**0.251**	0	0
Student-t (ρ, v)	0.267*		0.172	**28558.0**	**5.927**	0.346	0.001	0.001
Clayton (α)	0.291*	v = 23*	0.127	28627.5	5.942	0.738	0.092	0
Gumbel (δ)	1.191*		0.160	28616.5	5.939	0.584	0	0.210
Frank	1.618*		0.171	28590.2	5.934	0.361	0	0
				Panel (d) IBEX35 - Bitcoin				
Gaussian (ρ)	0.0037**		0.0024	13872.3	6.907	**0.314**	0	0
Student-t (ρ, v)	0.0037**		0.0024	**13871.9**	**6.905**	0.473	$5.1\ 10^{-9}$	$5.1\ 10^{-9}$
Clayton (α)	0.0145**	v =49**	0.0053	13872.8	6.908	0.324	$2.1\ 10^{-21}$	0
Gumbel (δ)	1.0011**		0.0017	13873.2	6.911	0.315	0	$2.1\ 10^{-08}$
Frank	0.0099**		0.0011	13872.3	6.907	0.315	0	0

AIC denotes the Akaike information criterion; BIC denotes the Bayesian information criterion; SSE denotes the root of the sum square error. In bold we remark the lowest values of BIC, AIC and SSE. (*) denotes significance at 1% level.

[9] The role of the Bitcoin as a hedge asset has been studied by Gkillas and Login, 2018; Kang et al., 2019; Klein et al., 2018; Feng et al., 2018; Bouri et al., 2017a; Bouri et al., 2017b; Briere et al., 2015; Eisl et al., 2015 and Dyhrberg, 2016.

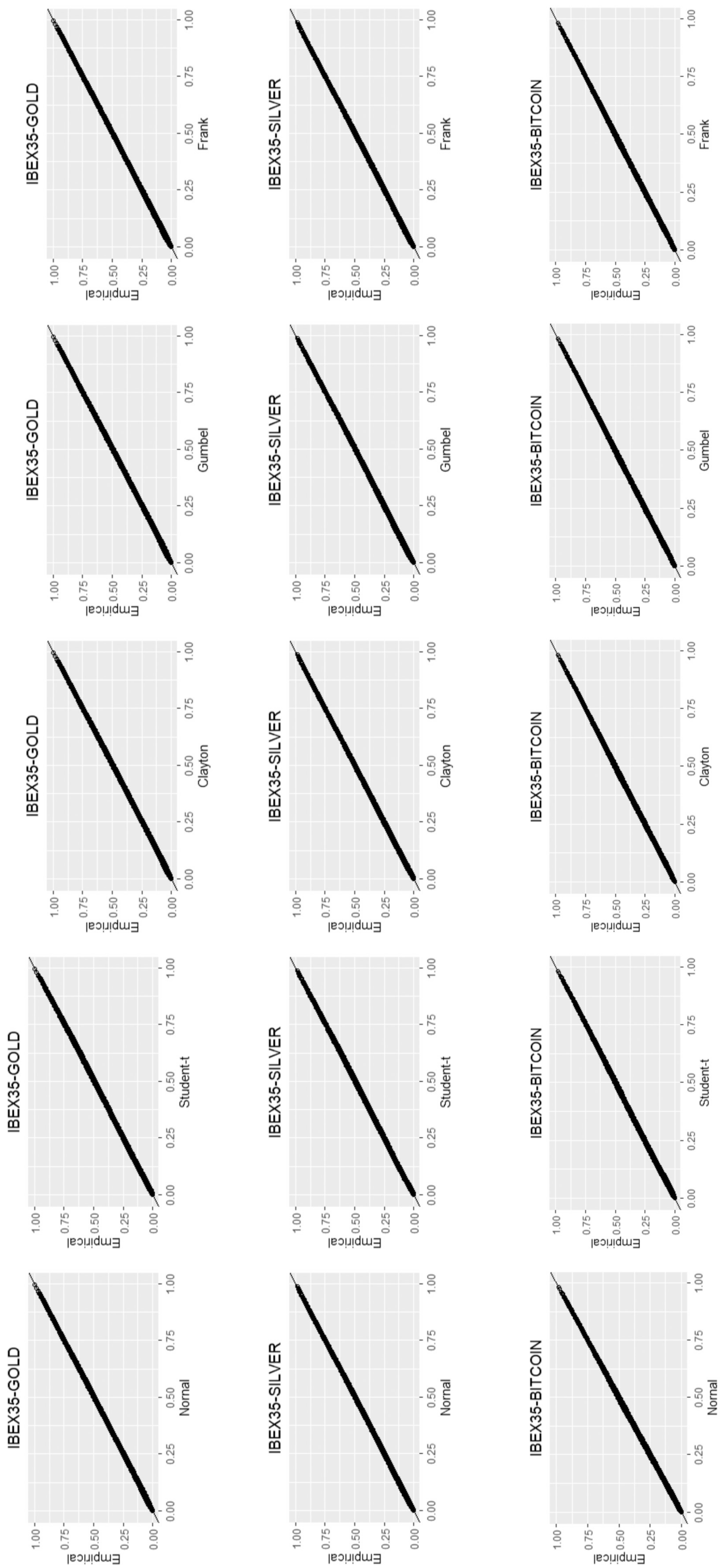

Figure 8: Scatter plots between the empirical copula and the parametric copula. **Commodities**

Furthermore, overall irrespectively of the copula functions used, all of them show null low tail dependence. This indicates that in extreme market conditions these assets may act as hedge assets against the risk of the Spanish stock market.

A visual inspection of the scatter plot of the data suggests that the fitting is quite good for all copula models not observing differences between them (Figure 8). According to the AIC and BIC criteria the student-t copula is the best in fitting data. However, according to the sum square errors, Clayton copula is the best in fitting data for Silver, Frank copula is the best for Gold and Gaussian copula is the best for Copper and Bitcoin.

To sum up, the European stock markets offer limited diversification possibilities for the Spanish investors as the dependence between the Spanish market and the European markets is quite high even in a normal market condition. The American markets offer greater diversifications possibilities than the European markets, especially the Brazilian and Argentina markets. The Asian markets outperform to the American markets offering higher diversifications possibilities even in extreme market conditions. The assets negotiated in the Shanghai market may be considered hedge assets instead of diversifier assets; this is also showed with the Gold and the Bitcoin. The Copper acts as a diversifier asset in a normal market condition but it may change its roll in an extreme market condition being a hedge asset in this case.

To last, comparing the correlations estimated (Table 4) with the results obtained with the copula analysis (Table 7, 8, 9, 10) we observe that degree dependence derived from the former is notably higher than the suggested by the copula analysis. This may be due the fact that correlation coefficient does not consider conditional heteroscedasticity so that correlations will be biased upwards (see Forbes and Rigobon, 2002).

4.4. Analyzing the time-varying dependence

The dependence between financial markets may change along the time making especially strong in a crisis period when the market volatilities increase. This is known as contagion effect. Forbes and Rigobon (2002) define contagion as the significant increase in cross-market correlations between any two markets from pre-crisis period to crisis period. In this paper, we study the dynamic dependence or contagion effect between two markets though a time-varying copula analysis.

The analysis carried out in the previous section (4.3) shows that, overall, all copula models fit well the data analysed. However, although the differences seem minimum the information criteria (AIC and BIC) and the sum square errors (SSE) point to the Elliptical copula model as the best in fitting data. Just only in the case of SSE both criteria point to Clayton as the best performing. Thus, for contagion effect study we focus in analysing the time-varying Student-t copula parameter which provides the best fitting in all cases except for SSE and IPC indexes. We analyse the time-varying Clayton copula parameter for SSE index and time-varying Normal copula parameter for IPC index[10].

To estimate daily copula parameters we use the equations (7), (8) and (9). The parameter estimations are reported in Table 11. Overall, the parameter estimations are all positive. In addition, for all copula models we observed that BIC and AIC values are smaller than those given by constant copula models which mean that the time-varying copula models outperform the constant copula model.

Although the copula parameters of the Student-t copulas are directly comparable, because of the similarity with the results reported in the above section, we focus our study on Kendall's τ associated with these parameters. Table 12 reports the mean Kendall's τ in three periods: pre-crisis (2000-2007), Global Financial crisis (2008-2010) and post-crisis (2011-2019).

[10] In any case, as all copula models provide similar parameter estimations, so that the selection copula model seems not be very relevant in this study.

Table 11: Time-varying copula specification and estimation

Index	Copula	ω	β	a	ν	AIC	BIC
Panel (a) European stock indices market							
CAC40	T-student	2.631	0.161	0.122	7	6613.4	1.336
FTSE100	T-student	1.955	0.127	0.086	7	3969.7	0.813
DAX	T-student	2.303	0.143	0.111	6	5300.2	1.075
Panel (b) American stock indices market							
S&P500	T-student	1.158	0.024	0.007	10	1490.4	0.307
MERVAL	T-student	0.895	0.200	0.060	13	654.5	0.137
IBOVESPA	T-student	0.964	0.155	0.063	15	759.7	0.157
IPC	Normal	-0.362	-0.591	-0.585	-	14861.4	3.083
Panel (c) Asian stock indices market							
NIKKEI	T-student	0.566	0.042	0.007	43	334.0	0.070
HSI	T-student	0.763	0.081	0.034	25	513.4	0.106
KOSPI	T-student	0.485	0.043	-0.016	35	345.3	0.071
SSE	Clayton	0.079	0.060	-0.007	-	367.7	0.064
Panel (d) Commodities							
Gold	T-student	-0.040	0.527	-0.000	20	65.6	0.012
Silver	T-student	0.095	0.171	-0.005	22	47.3	0.008
Copper	T-student	0.467	0.208	-0.017	23	383.6	0.078
Bitcoin	T-student	0.017	-0.003	-2.2 e-05	49	1.4	0.003

The table provides information on maximum likelihood parameter for the Normal copula (equation, 7) Student-t copula model (equation, 8) and Clayton copula model (equation, 9).

Table 12: Average of the dependence copula parameter and kendall's tau

	Before GFC 2000-2007		During GFC 2008-2010		After GFC 2011-2019	
	Copula parameter	Kendall's tau	Copula parameter	Kendall's tau	Copula parameter	Kendall's tau
Panel (a) European stock indices market						
CAC40	0.861	0.661	**0.865**	**0.666**	0.862	0.662
FTSE100	0.753	0.543	**0.760**	**0.550**	0.752	0.542
DAX	0.823	0.616	**0.827**	**0.621**	0.823	0.616
Panel (b) American stock indices market						
S&P500	0.528	0.354	**0.531**	**0.356**	0.529	0.355
MERVAL	0.372	0.243	**0.397**	**0.260**	0.377	0.246
IBOVESPA	0.411	0.270	**0.426**	**0.281**	0.408	0.268
IPC	**0.362**	**0.236**	0.323	0.210	**0.379**	**0.248**
Panel (c) Asian stock indices market						
NIKKEI	0.271	0.175	**0.272**	**0.176**	0.271	0.175
HSI	0.345	0.224	**0.348**	**0.226**	0.345	0.224
KOSPI	0.243	0.156	**0.243**	**0.156**	0.241	0.155
SSE	0.060	0.029	**0.067**	**0.032**	0.064	0.031
Panel (d) Commodities market						
Gold	-0.026	-0.017	**-0.002**	**-0.001**	-0.039	-0.025
Silver	0.053	0.034	**0.075**	**0.048**	0.054	0.034
Copper	0.250	0.161	**0.284**	**0.184**	0.256	0.165
Bitcoin	--	--	--	--	0.006	0.004

Note: We use Student-t copula model for all the asset except for IPC (Normal copula model) and SSE (Clayton copula model). We highlight in grey the period in which the dependence increases.

Figure 9 illustrates the dynamic of Kendall's τ for the European stock market considered. The first thing to emphasize is that overall, Kendall's τ are very stable, remaining close to the value estimated by the constant copula. Therefore, we do not observe a changing trend in the dependence structure estimated between these markets throughout the period although in average terms it becomes higher between 2008 and 2010, coinciding with the global financial crisis (see Table 12). In that period, some specific jumps in the level of dependency are observed which are common to all markets. It suggests the existence of contagion effect. These results confirm that the European markets offer limited diversification possibilities for the Spanish investor, especially in crisis period when punctually the dependence become higher.

Figure 10 illustrates the dynamics of the Kendall's τ derived from the time-varying Student-t copula for the American markets. In the case of the US stock market, the dependence derived from the time-varying Student-t copula is quite stable, without remarkable changes during the GFC. For the Mexican stock market (IPC) the degree of dependence is highly volatile, moving between -0,16 and 0.40, and dependence degree during the GFC become lower than before and after GFC (Table 12). These results reveal that the Mexican stock market may provide higher diversification possibilities than these derived from the analysis static which suggest a Kendall's τ of 0.29.

On the other hand, the dependence degrees between the Spanish stock market and the Brazilian and Argentine stocks markets are also volatile but in both cases the deviations with respect to the mean are somewhat lower. Here we also observed that the dependence increases during the GFC, confirming the existence of contagion effect (see, Table 12).

Figure 11 illustrates the dynamics of the Kendall's τ derived from the time-varying copula for the Asian markets. We observe that the dependence between IBEX35 and these markets was very low along the whole period, before, during and after financial crisis. For instance, the dependence with the Japan market was 0.175 before crisis, 0.176 during the crisis and 0.175 after crisis. Thus, the assets negotiated in these markets may be considered adequate to diversify against the Spanish stock performance. These results corroborate those obtained in the copula analysis reported in previous section which indicated low tail dependence.

The most striking case is observed for the Shanghai market which kept on a null dependence along the whole period although increase slightly during the crisis (Table 12). The assets negotiated in this market may be a hedge asset rather than a good diversifier. Regarding the contagion effect, the results confirm those obtained in the literature which point that in spite of the strong increase of the trade with China, the contagion between the European stock markets and the Shanghai market is still very reduced (Rajwani et al., 2019 and Nugyen et al., 2017).

Figure 12 illustrates the dynamics of the Kendall's τ derived from the time-varying Student-t copula for the Commodities markets. Regarding to the Gold market we observe that the dependence degree is very volatile. The 63% of the days, the Kendall's τ is negative, which means that mostly Gold acts as a hedge asset, but there is still a significant number of days in which this asset behavior as a diversifying asset (37%). In addition, we observed that during the GFC the dependence degree increased. In this period the Kendall's τ was positive the 50% of the days, reaching record values in the sample. This means that also in this market there was a contagion effect. Regarding to the Silver market, the dependence degree moved between -0.1 and 0.1, although mostly is positive. Just only the 7% of the days the Kendall's tau was negative. This means that the Silver is mostly a diversifier asset. As in the case of Gold, the dependence became higher in the GFC period. Similar results are observed for Copper. This asset can be considered a diversifier asset in normal market conditions but in crisis period the dependence increase lightly.

The most striking case is found for the Bitcoin. This asset keeps on a null dependence along the whole period, without punctual increase in dependence in the whole period.

The results presented in this section corroborate some of those obtained in the static analysis and spell out some others. The European stock markets offer limited diversification possibility for the Spanish investors as the dependence between the Spanish stock market and these markets is quite high even in a normal market conditions. In addition, in extreme market conditions the dependency is punctually higher.

The American markets offer higher diversification possibilities than the European market but the diversification may work somewhat worse in an extreme market condition as the dependence in these periods increases punctually. This is observed specially in the Brazilian and Argentine markets. To emphasize that the dependence with the Mexican market decreased in the GFC period. On the other hand, the Asian markets outperform to the American markets offering higher diversification possibilities even in extreme market conditions. Only in some days the dependence increases punctually. To last, the assets negotiated in the Shanghai market may be considered hedge assets instead of diversifier assets; this is also showed by the Bitcoin, Gold and Silver, although the role of these two last assets is highly volatile.

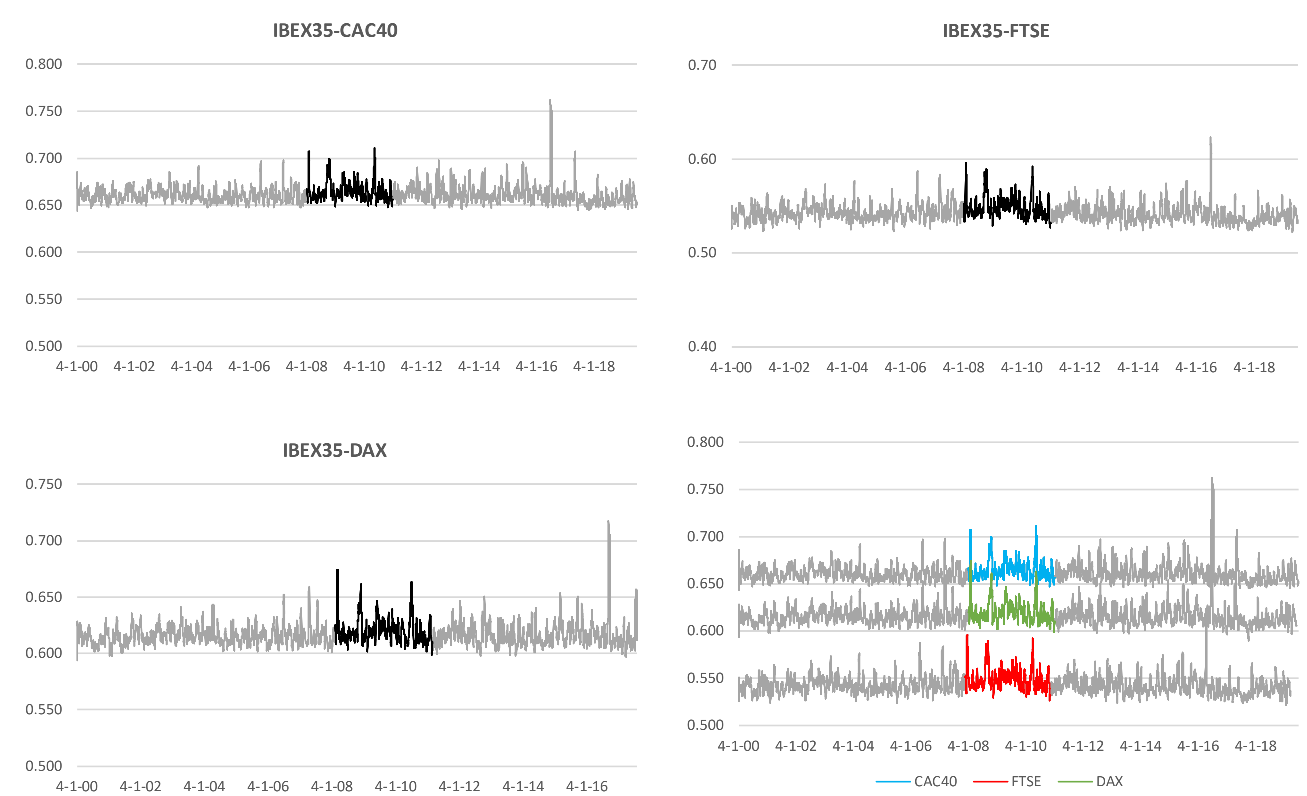

Figure 9 illustrates the dynamic of the daily Kendall's Tau estimated between the IBEX35 and some European stock indexes. In bold we remark the Global Financial Crisis (January, 2008 and December, 2010)

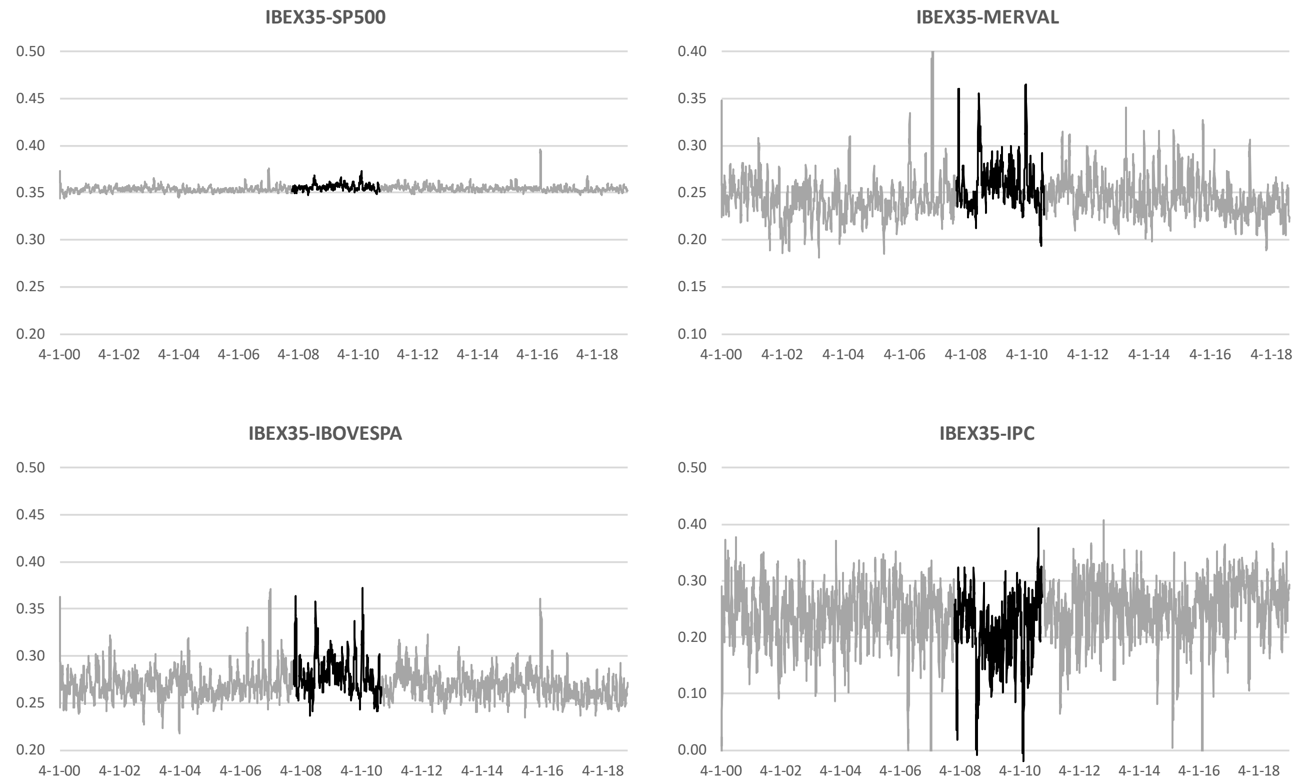

Figure 10 illustrates the dynamic of the daily Kendall's Tau estimated between the IBEX35 and some American stock indexes. In bold we remark the Global Financial Crisis (January, 2008 and December, 2010)

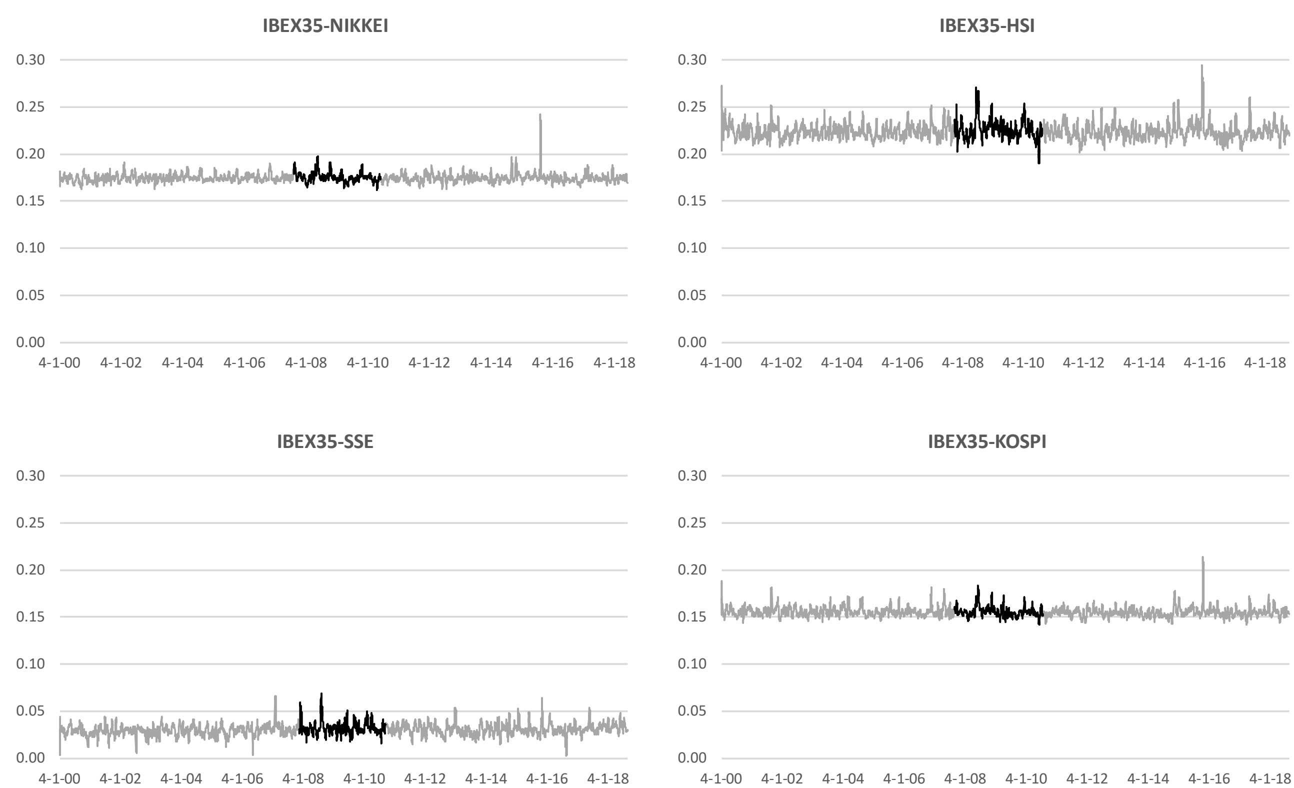

Figure 11 illustrates the dynamic of the daily Kendall's Tau estimated between the IBEX35 and some Asian stock indexes. In bold we remark the Global Financial Crisis (January, 2008 and December, 2010)

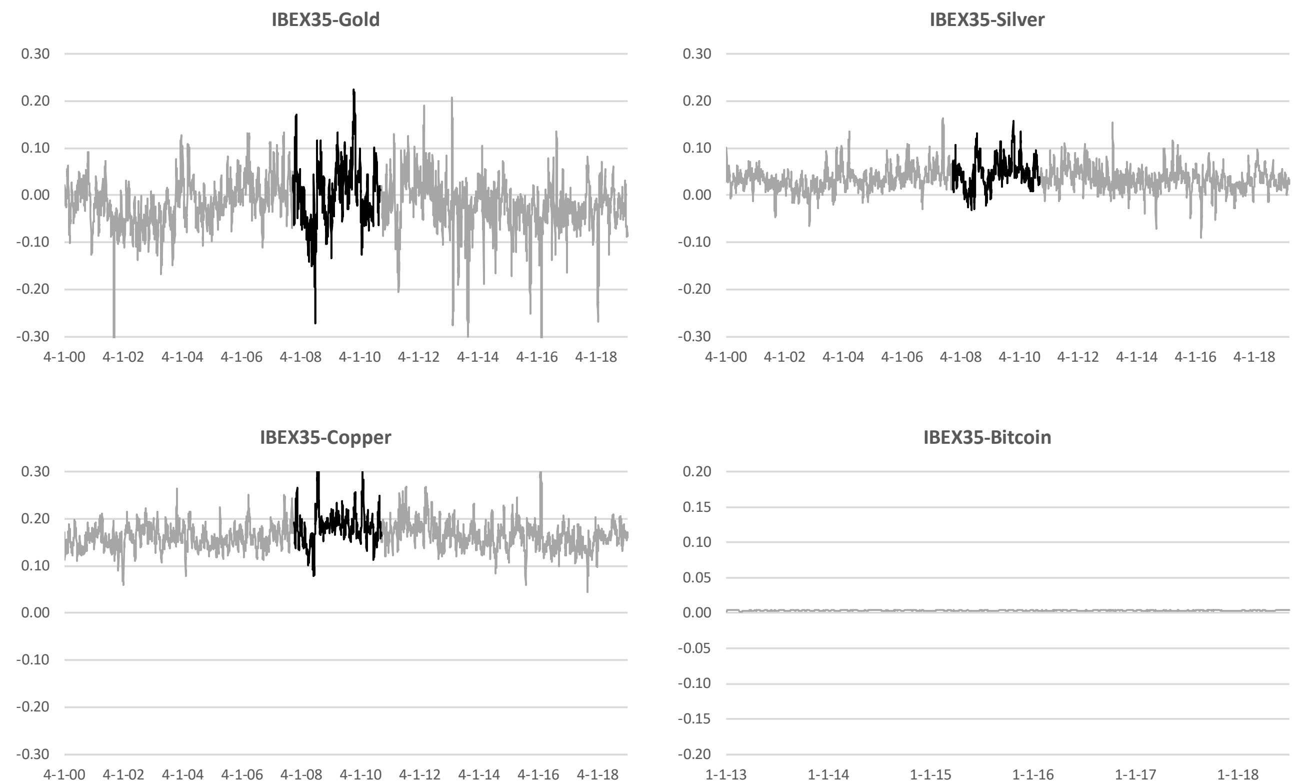

Figure 12 illustrates the dynamic of the daily Kendall's Tau estimated between the IBEX35 and some commodity indexes. In bold we remark the Global Financial Crisis (January, 2008 and December, 2010)

The time-varying copula models estimated in this paper might be used to estimate the aggregate risk of a portfolio of a pair of the indexes considered. The advantage of using time varying copula model is that the aggregate risk can be calculated without assuming a perfect correlation between markets which is pretty usual in practice. Let be *X* the return of a portfolio of two assets, *Y* and Z. Using time-varying copula model, the VaR of the portfolio can be calculated as follow:

$$(aggVaR^{\alpha}_{t+1})^2 = \left(VaR^{\alpha}_{Y,t+1}, VaR^{\alpha}_{Z,t+1}\right)\begin{pmatrix}1 & \tau \\ \tau & 1\end{pmatrix}\begin{pmatrix}VaR^{\alpha}_{Y,t+1} \\ VaR^{\alpha}_{Z,t+1}\end{pmatrix}$$

$$aggVaR^{\alpha}_{t+1} = \left(\left(VaR^{\alpha}_{Y,t+1}\right)^2 + 2\tau VaR^{\alpha}_{Z,t+1} VaR^{\alpha}_{Y,t+1}, + \left(VaR^{\alpha}_{Z,t+1}\right)^2\right)^{1/2}$$

where τ captures the dependence between the marginal, *Y* and *Z*. In our study, *X* and Y represent the indexes's returns. This means that the aggVaR forecast above incorporates interactions between different returns by introducing their dependence measures. Note that the estimated aggVaR forecast may be obtained by using the individual estimative VaR forecasts and the estimate of dependence measures.

If $\tau = 1$, as it is supposed in many cases, the VaR of the portfolio at time $(t + 1)$ is calculated as:

$$aggVaR^{\alpha}_{t+1} = VaR^{\alpha}_{Y,t+1} + VaR^{\alpha}_{Z,t+1}$$

but if $\tau < 1$ then, the above equation overestimate risk. So, in order to quantify market risk of a portfolio properly, it is very important to know the dependence structure between the assets of the portfolio.

4.5. Conclusions

In this study we investigate the dependence structure between the Spanish stock market, represented by the IBEX35 Index, and some international financial markets, both stocks and commodities. To assess this study we use copula analysis which appropriately describes the dependence structure between financial assets. The aim is twofold: (i) to understand the relationship between these markets and to establish the importance of copula analysis with respect to the linear correlation coefficient in understanding such relationship and (ii) to analyze the possibility of diversification and hedge that these markets offer to the Spanish investors and vice versa.

The stock markets considered are: French, English, German, American (US), Argentine, Mexican, Brazilian, Japanese, Korean, Hong Kong and Chinese, this last represented by the Shanghai index. The commodities included in this study are: Gold, Silver, Copper and Bitcoin. As the dependence structure may change along the time we conduct this study in static and dynamic terms. By studying the dependence in dynamic terms we want to know whether the dependence structure increases significantly in a crisis period, when the diversification is more necessary.

The European stock markets offer limited diversification possibility for the Spanish investors as the dependence between the Spanish stock market and these markets is quite high even under normal market conditions. In addition, in extreme market conditions the dependence is punctually higher.

The American markets offer higher diversification possibilities than the European markets but the diversification may work somewhat worse in an extreme market condition as the dependence in these periods increases punctually. This is observed specially in the Brazilian and Argentine markets. By other hand, the Asian markets outperform to the American markets offering higher diversification possibilities even in extreme market conditions. Besides, the assets negotiated in the Shanghai market may be considered hedge assets instead of diversifier assets; this is also showed by the Bitcoin and Gold although the role of this last assets is highly volatile. In some periods he can act as a hedge asset but in others they just act as diversifier assets. These results provide useful information for those who seek to actively diversify their international portfolios and to manage their worldwide assets.

5. References

Baig, T. and Goldfajn, I., 1999. Financial Market Contagion in the Asian Crisis, IMF Staff Papers. Volume 46, 167–195.

Baur, D.G., and Lucey, B.M., 2010. Is Gold a Hedge or a Safe Haven? An analysis of Stocks Bonds and Gold, The Financial Review. Volume 45, 217-229.

Baur, D., 2012. Asymmetric volatility in the gold market, Journal of Alternative Investments. Volume 14, 26–38.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., and Hagfors, L. I., 2017b. On the hedge and safe haven properties of Bitcoin: is it really more than a diversifier? Finance Research Letters. Volume 20,192-198.

Bouri, E., Jalkh N., Molnár, P., Roubaud, D., 2017a. Bitcoin for energy commodities before and after the December 2013 crash: diversifier, hedge or safe haven? Applied Economics. Volume 49(50), 5063-5073.

Calvo, G.A., L., Leiderman, L. and Reinhart, C.M., 1996. Inflows of Capital to Developing Countries in the 1990s, Journal of Economic Perspectives. Volume 10(2), 123-139.

Case, B., Yang, Y., and Yildirin, Y., 2012. Dynamic Correlations Among Asset Classes: REIT and Stock Returns, The Journal of Real Estate Finance and Economics. Volume 44, 298–318.

Changquing, L., Chi, X., Cong, Y., and Yan, X., 2015. Measuring financial market risk contagion using dynamic MRS-Copula models: The case of Chinese and other international stock markets, Economic Modelling. Volume 51, 657-671

Cherubini, U., and Luciano, E., 2001. Value at risk Trade off and Capital Copulas, Economic Notes. Volume 30, 235-256

Das, S., 2016. Cointegration of Bombay Stock Exchange with Major Asian Markets—A Copula Approach, Global Business Review. Volume 13(3), 566-581

Ding, Z., Granger, C.W.J., and Engle, R.F., 1993. A long memory property of stock market returns and a new model, Journal of Empirical Finance. Volume 1, 83-106.

Dyhrberg, A. H., 2016. Bitcoin, gold and the dollar – A GARCH volatility analysis, Finance Research Letters. Volume 16, 85-92

Engle, R., 1982. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica. Volume, 50(4) 987-1007.

Eisl, A., Gasser, S., Weinmayer, K., 2015. Caveat Emptor: Does Bitcoin Improve Portfolio Diversification? Available at SSRN: 2408997. http://dx.doi.org/10.2139/ssrn.2408997

Fang, Y., Madsen, L., and Liu L., 2007. Comparison of Two Methods to Check Copula Fitting, IAENG International Journal of Applied Mathematics. Volume, 44(1), 1-9.

Feng, W., Wang, Y., Zhang, Z., 2018. Can Cryptocurrencies Be a Safe Haven: A Tail Risk Perspective Analysis, Applied Economics. Volume 50(44), 4745-4762.

Fermanian, J.D., 2005. Goodness-of-fit tests for copulas, Journal of Multivariate Analysis. Volume 95, 119–152.

Forbes, K., and Rigobon, R., 2002. No Contagion, Only Interdependence: Measuring Stock Market Comovements, The Journal of Finance. Volume, 57(5), 2223-2261.

Frey, R., and McNeil, A., 2003. Dependent Defaults in Models of Portfolio Credit Risk, Journal of Risk. Volume 6(1): 59-92.

Gkillas, K., Longin, F., 2018. Is Bitcoin the New Digital Gold? Evidence From Extreme Price Movements in Financial Markets. Evidence From Extreme Price Movements in Financial Markets (October 20, 2018). Available at SSRN: https://ssrn.com/abstract=3245571

Gregoire, V., Genest, C., and Gendron, M., 2008. Using copulas to model price dependence in Energy Markets, Energy Risk.Volume, 6(3), 62-68.

He, X.D., Jin, H. and Zhou, X.Y., 2015. Dynamic portfolio choice when risk is measured by weighted VaR, Mathematics of operations research. Volume 40(3).

Hon, M. T., Strauss, J. K. and Yon S-K, 2007. Deconstructing the Nasdaq bubble: A look at contagion across international stock markets, Journal of International Financial Markets, Institutions and Money. Volume 17(3), 213-230.

Horta, P., Mendes, C. and Vieira, I. 2010. Contagion effects of the subprime crisis in the European NYSE Euronext markets, Portuguese Economic Journal. Volume 9, 115–140.

Hussain, S. I. and Li, S., 2018. The dependence structure between Chinese and other major stock markets using extreme values and copulas, International Review of Economics & Finance. Volume 56, 421-437.

Joe, H., 1997. Multivariate Models and Dependence Concepts, Chapman and Hall, London.

Jondeau, E., and Rockinger, M., 2006. The Copula-GARCH model of conditional dependencies: An international stock market application, Journal of International Money and Finance. Volume 25(5), 827-853.

Junker, M., Szimayer, A., and Wagner, N., 2006. Nonlinear Term Structure: Dependence: Copula Functions, Empirics and Risk Implications, Journal of Banking and Finance. Volume 30, 1171-1199.

Kang, S.H., Mclver, R.P., Arreola, J., 2019. Co-movements between Bitcoin and Gold: A wavelet coherence analysis, Physica A: Volume 536, 1-9.

Kenourgios, D., Samitas, A. and Paltalidis, N., 2010. Financial crises and stock market contagion in a multivariate time-varying asymmetric framework, Journal of International Financial Markets, Institutions & Money. Volume 21(1), 92-106.

King, M. A. and Wadhwani, S., 1990. Transmission of Volatility between Stock Market, The Review of Financial Studies. Volume 3(1), 5–33

Klein, T., Hien, P.T. and Walther, T., 2018. Bitcoin is not the New Gold – A comparison of volatility, correlation, and portfolio performance, International Review of Financial Analysis. Volume 59, 105-116, Luciano, E., and Marena, M., 2002. Copula as a new tool in financial modelling, Operational Research: An International Journal. Volume 2: 139-155.

Mandelbrot, B., 1963 New Methods in Statistical Economics, The Journal of Political Economy. Volume 71(5), 421-440.

Meucci, A., 2010. Managing Diversification, Bloomberg Education & Quantitative Research and Education Paper, 74-79.

Nguyen, C., Ishac, B.M., and Henri, D., 2017. Are Vietnam and Chinese stock markets out of the US contagion effect in extreme events? Physica A: Statistical Mechanics and its Applications. Volume 480(15) 10-21

Patton, A., 2006. Modelling asymmetric exchange rate dependence, International Economic Review. Volume 47(2), 527-556.

Rajwani, S. and Kumar, D., 2019. Measuring Dependence Between the USA and the Asian Economies: A Time-varying Copula Approach. Global Business Review, Volume 27.

Reboredo, J.C., 2011. How do crude oil prices co-move? A copula approach. Energy Economics. Volume 33(5), 948-955

Rong, N., and Trück, S., 2014. Modelling the Dependence Structure Between Australian Equity and Real Estate Market – a Copula Approach, Australian Accounting, Business and Finance Journal. Volume 8(5): 93-113.

Samitas, A. and Tsakalos, I., 2013. How can a small country affect the European economy? The Greek contagion phenomenon, Journal of International Financial Markets, Institutions and Money. Volume 25, 18-32

Syriopoulos, T. and Roumpis, R., 2009. Dynamic correlations and volatility effects in the Balkan equity markets, Journal of International Financial Markets, Institutions and Money. Volume 19(4), 565-587.

Yang, J., Zhou, Y. and Leung, W.K., 2012. Asymmetric Correlation and Volatility Dynamics among Stock, Bond, and Securitized Real Estate Markets. The Journal of Real Estate Finance and Economics. Volume 45, 491–521.

Wang, Y-Q, Shi-wen, L. 2011. Financial market openness and risk contagion: A time-varying Copula approach. College of Finance, Zhejiang Gongshang University, 31(4): 778-784.

Weng, X., Wei, Y. and Huang, D., 2012. Measuring contagion between energy market and stock market during financial crisis: A copula approach. Energy Economics Volume 34(5) 1435-1446